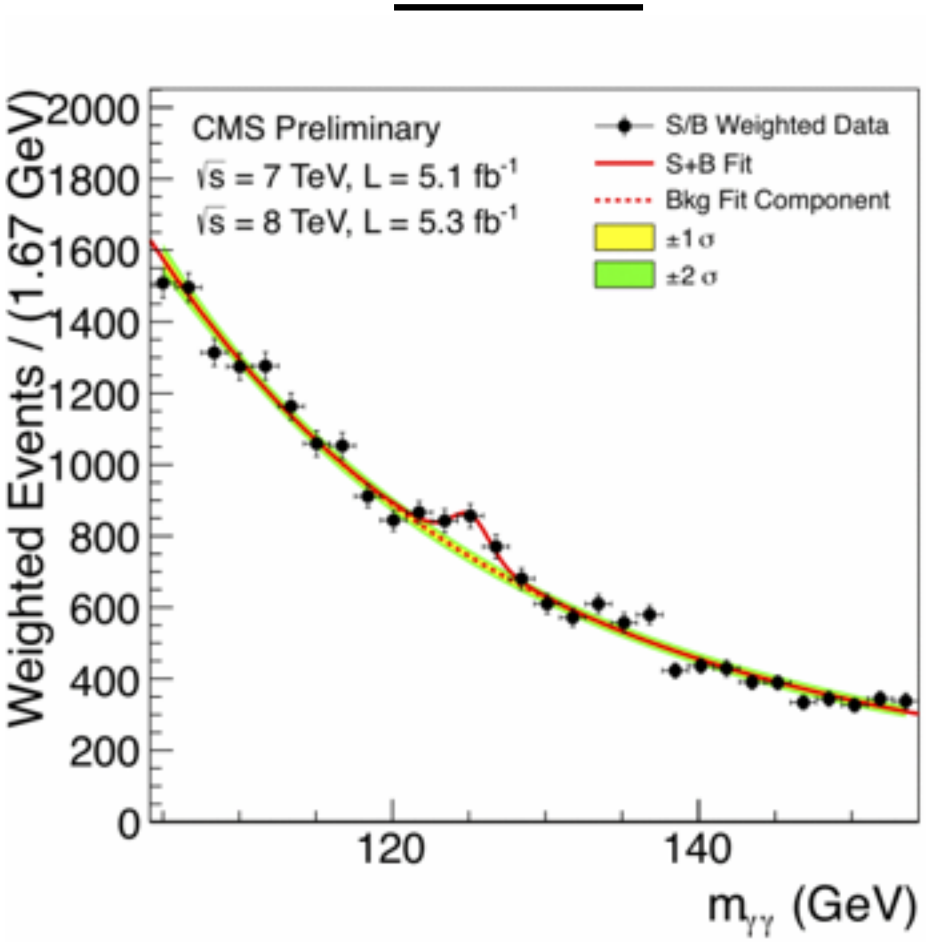
Naturalness and the Weak Gravity Conjecture

Clifford Cheung

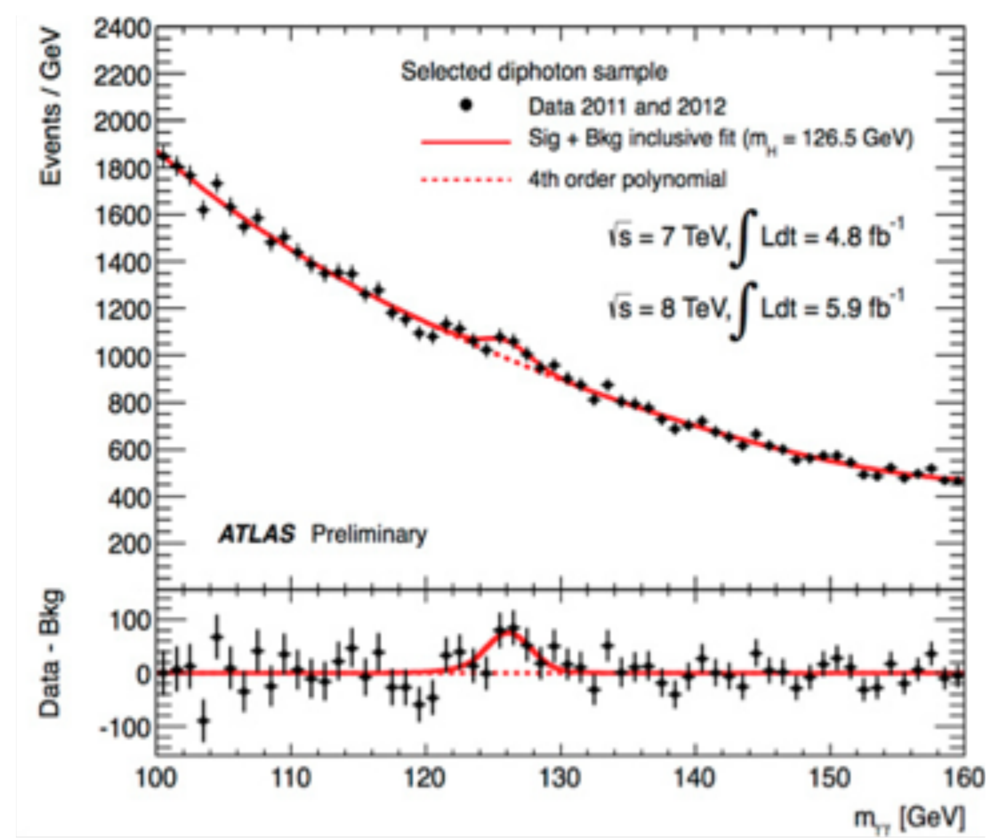
C. Cheung, G. N. Remmen (1402.2287, 14xx.xxxx)

LHC has discovered a new scalar.

<u>CMS</u>



<u>ATLAS</u>



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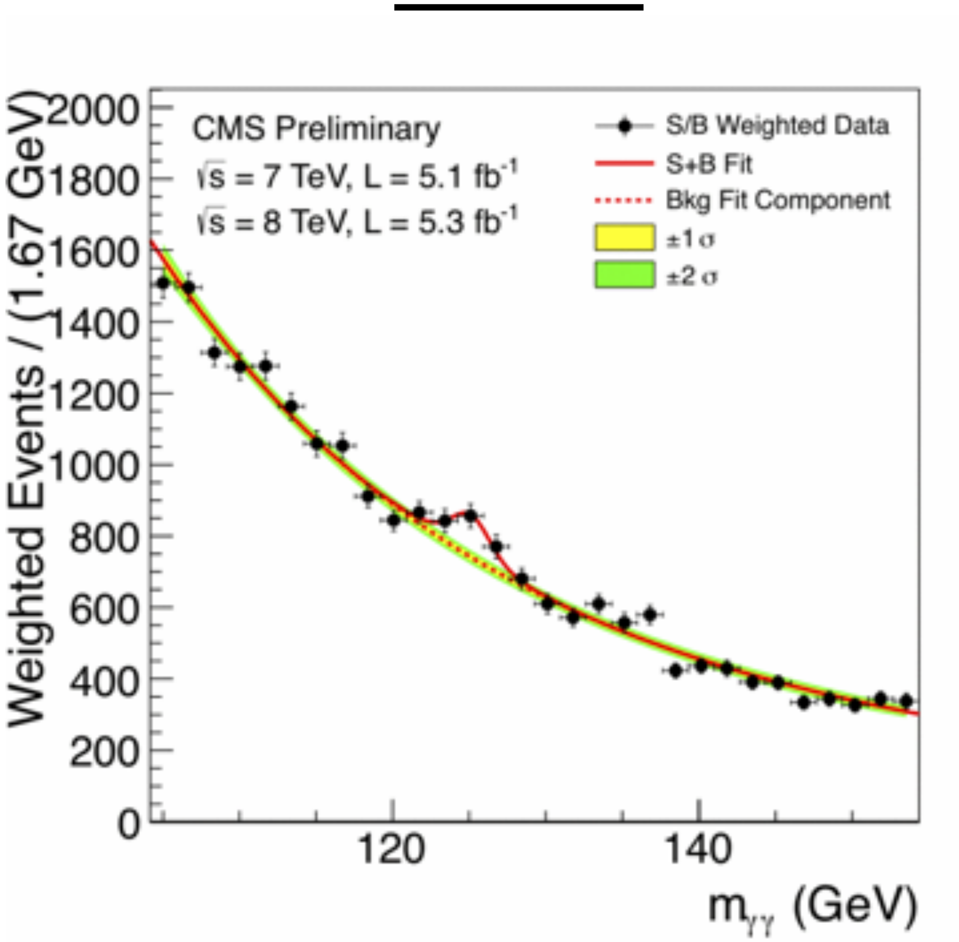

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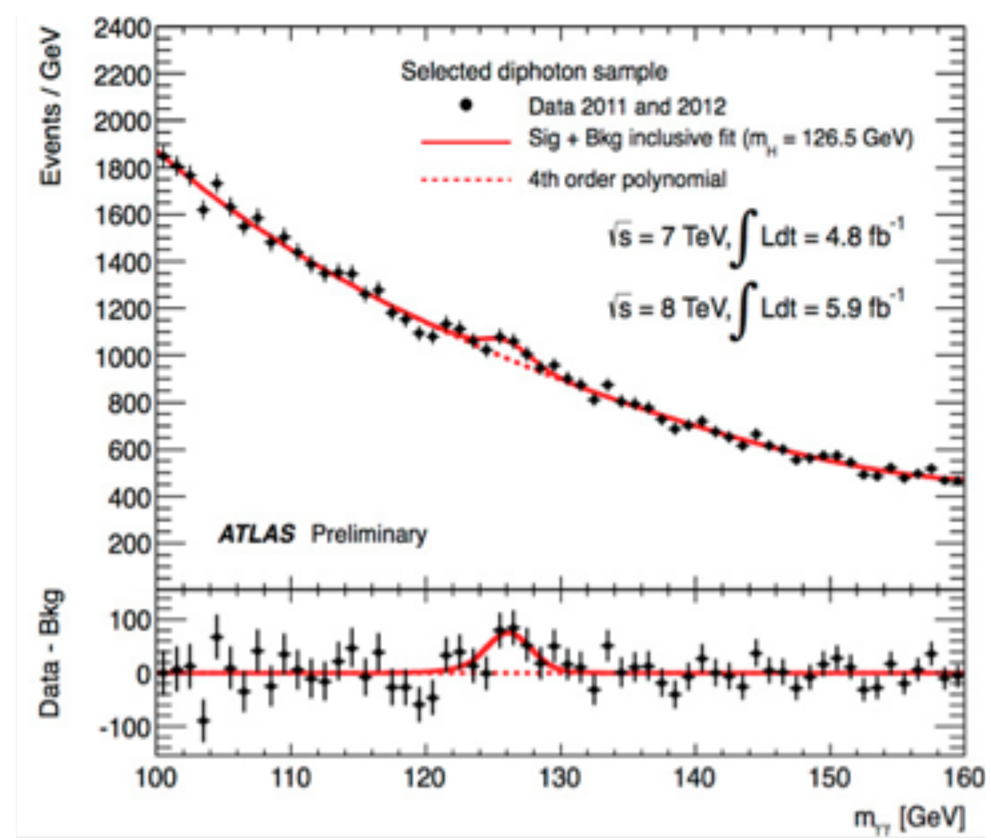

Where are the “naturalons”?

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- meso-tuning ⟵ (acceptance)

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weak gravity conjecture

weak gravity conjecture (WGC)

(Arkani-Hamed, Motl, Nicolis, Vafa)

A long-range U(1) coupled consistently to gravity *requires* a state with

$$q > m/m_{\rm Pl}$$

which is a non-perturbative, highly non-trivial criterion for healthy theories. In short:

"Gravity is the weakest force."

evidence #1

The WGC is satisfied by a litany of healthy field and string theories.

For example, for SU(2) → U(1) gauge theory,

$$g > m_W / m_{\rm Pl} \xrightarrow{(m_W = gv)} m_{\rm Pl} > v$$

and similarly for the monopoles.

evidence #2

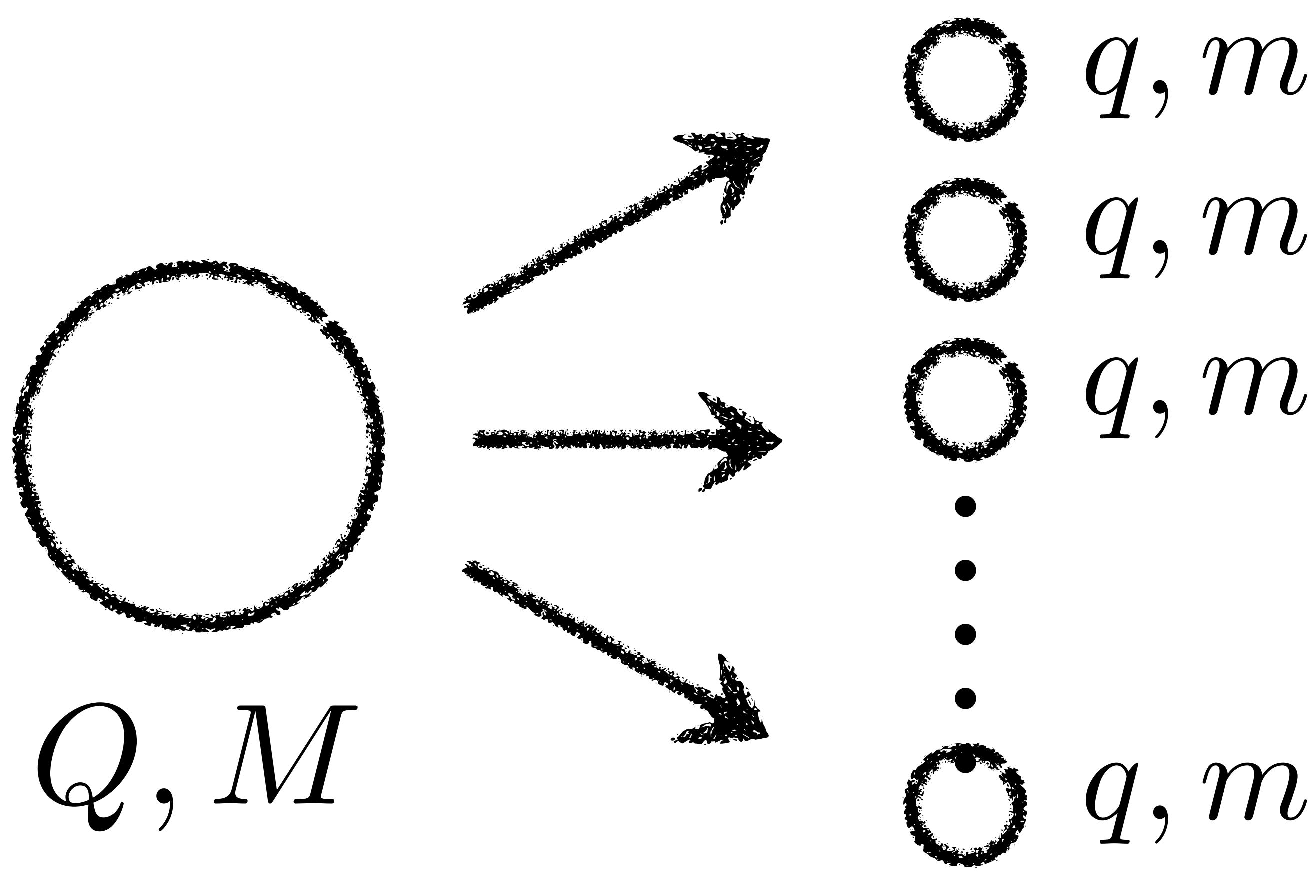
Without WGC, the $q \rightarrow 0$ limit can be taken to yield an exact global symmetry.

However, exact global symmetries are forbidden by black hole no-hair theorems.

Static black holes are labelled solely by their mass, spin, and charge.

evidence #3

The authors of the WGC justified it with a Gedanken experiment with black holes:



number of particles in final state $= \quad Q/q$ conservation of charge

total rest mass in final state $= \quad mQ/q < M$ conservation of energy

For an extremal black hole, $Q = M/m_{\rm Pl}$, so

$$q > m/m_{\rm Pl}$$

When the WGC criterion fails, extremal black holes are exactly stable.

In such a theory there will be a huge number of stable black hole remnants.

This yields serious pathologies:

- thermodynamic catastrophes
- tension with holography

The story is the same with many charged species. We define convenient notation:

$$z_i = q_i m_{\mathrm{Pl}}/m_i \qquad \text{(particle species i)}$$

$$Z = Q\, m_{\mathrm{Pl}}/M = 1 \qquad \text{(extremal black hole)}$$

So the WGC states there there must exist a particle species i for which:

$$z_i > 1$$

evidence #4 (new)

Failure of the WGC should yield pathologies which are visible at low energies.

Integrate out all but the photon and graviton:

$$\mathcal{L}_F = a_1 (F_{\mu\nu} F^{\mu\nu})^2 + a_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2$$

$$\mathcal{L}_{FR} = b_1 F_{\mu\nu} F^{\mu\nu} R + b_2 F_{\mu\rho} F_{\nu}{}^{\rho} R^{\mu\nu} + b_3 F_{\mu\nu} F_{\rho\sigma} R^{\mu\nu\rho\sigma}$$

$$\mathcal{L}_R = c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$$

Crucially, the coefficients of the effective action encode important information.

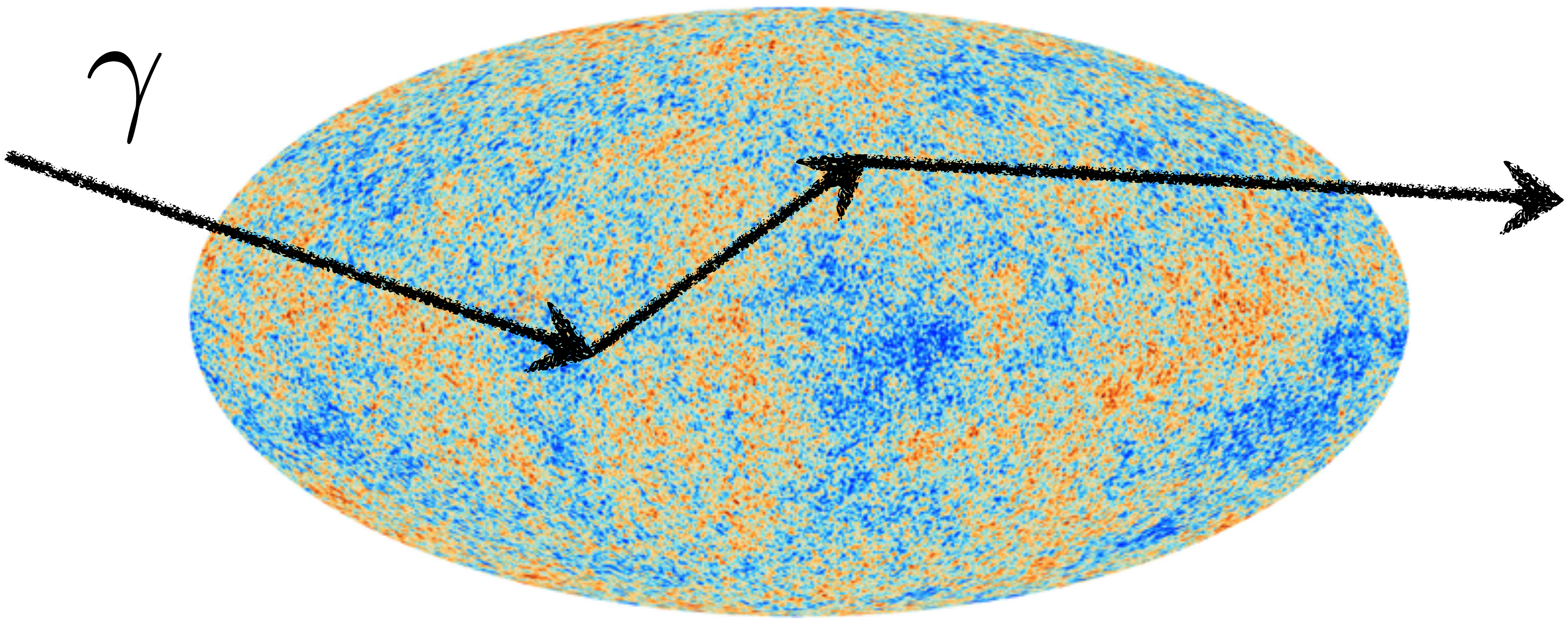
$$aF^4 \qquad a \sim z^4$$

$$bF^2 R \qquad b \sim z^2$$

$$cR^2 \qquad c \sim 1$$

How are a, b, c constrained at low energies?

Photon propagation is modified in a radiation dominated FRW universe.



Setting $\langle F^2 \rangle, \langle R \rangle \neq 0$, we find that photons are superluminal unless $z \gtrsim \mathrm{const}$!

naturalness and WGC

But we have ignored a crucial effect, which is that charges and masses are loop corrected!

$$q(\mu) > m(\mu)/m_{\rm Pl}$$

renormalized quantities

We should evaluate quantities at pole mass.

Note: WGC can bound a radiatively *unstable* quantity (mass) by a *stable* one (charge).

scalar QED

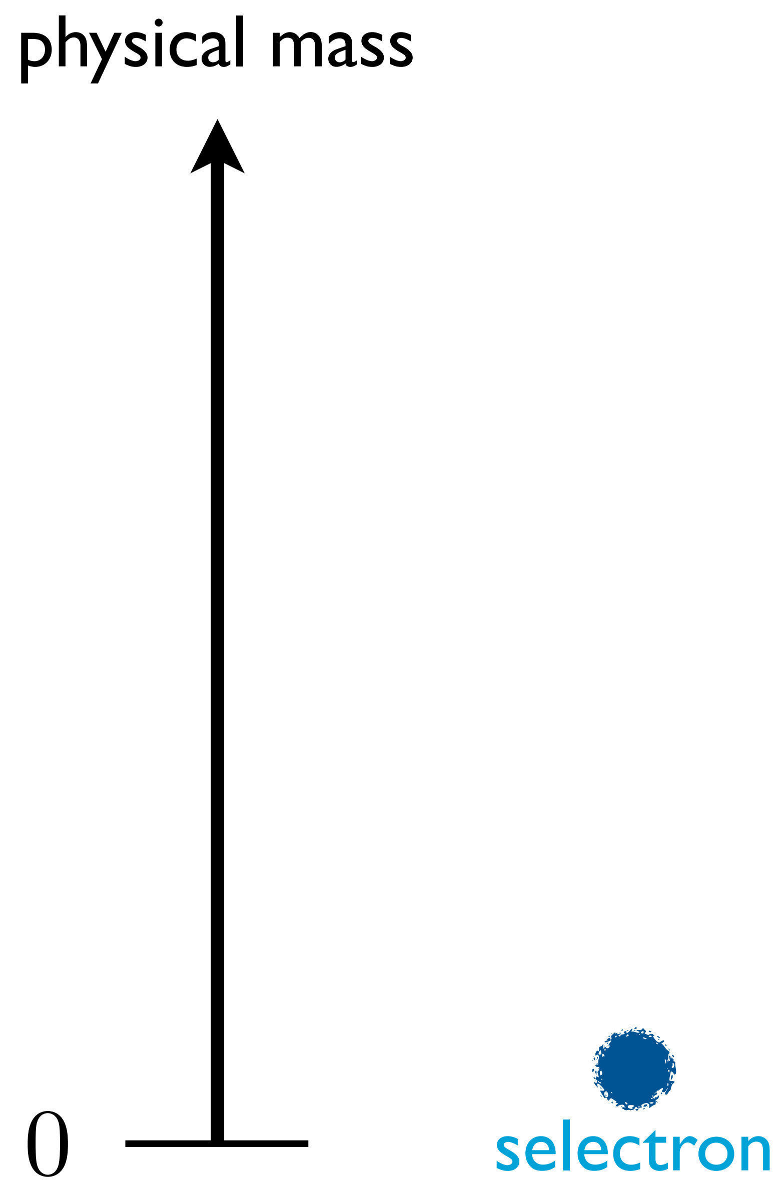
Take the very simplest case of a U(1) charged particle with a hierarchy problem:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu \phi|^2 - m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4$$

where the "selectron" has charge q:

$$D_\mu = \partial_\mu + iqA_\mu$$

There is fundamental tension between naturalness and the WGC.



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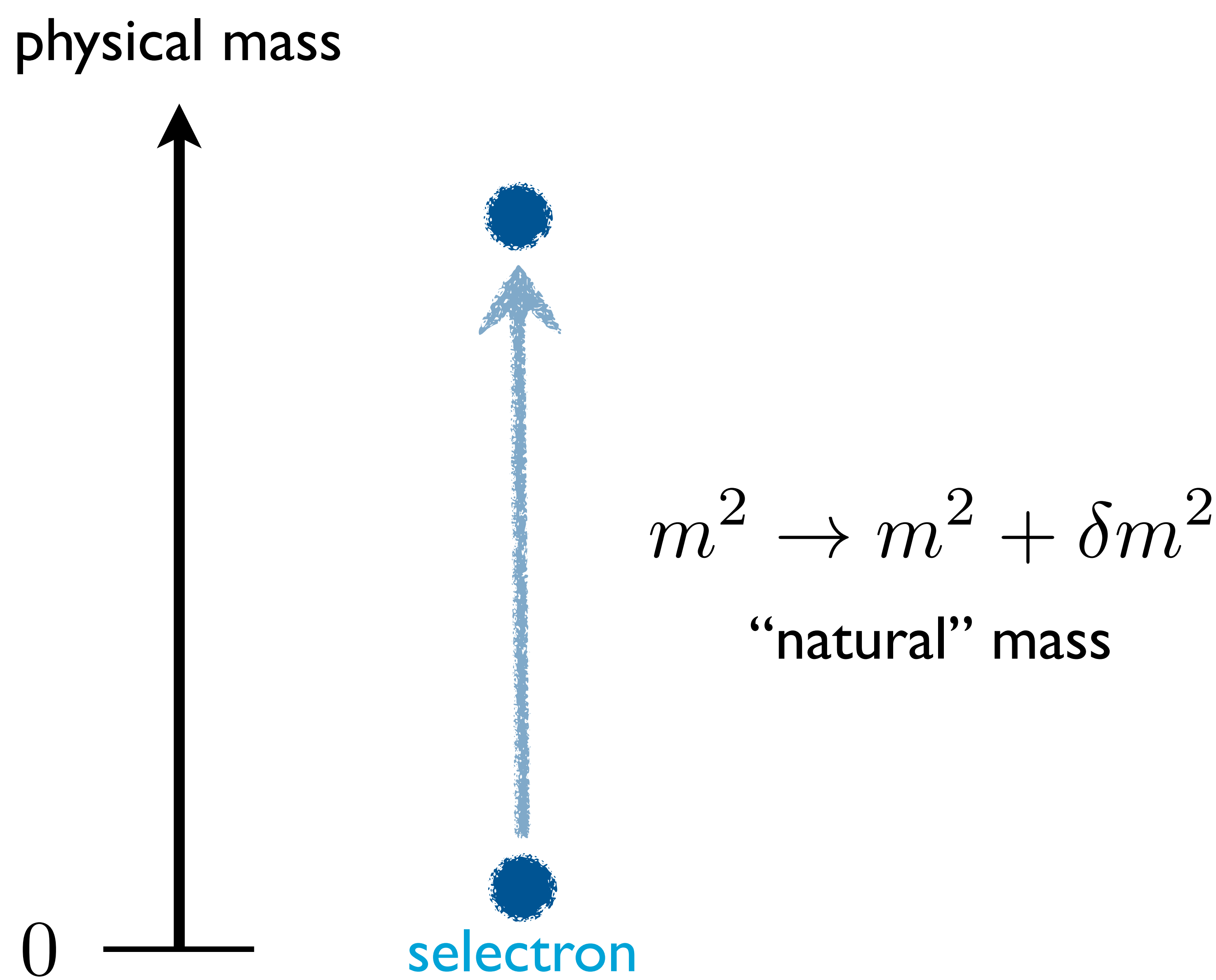

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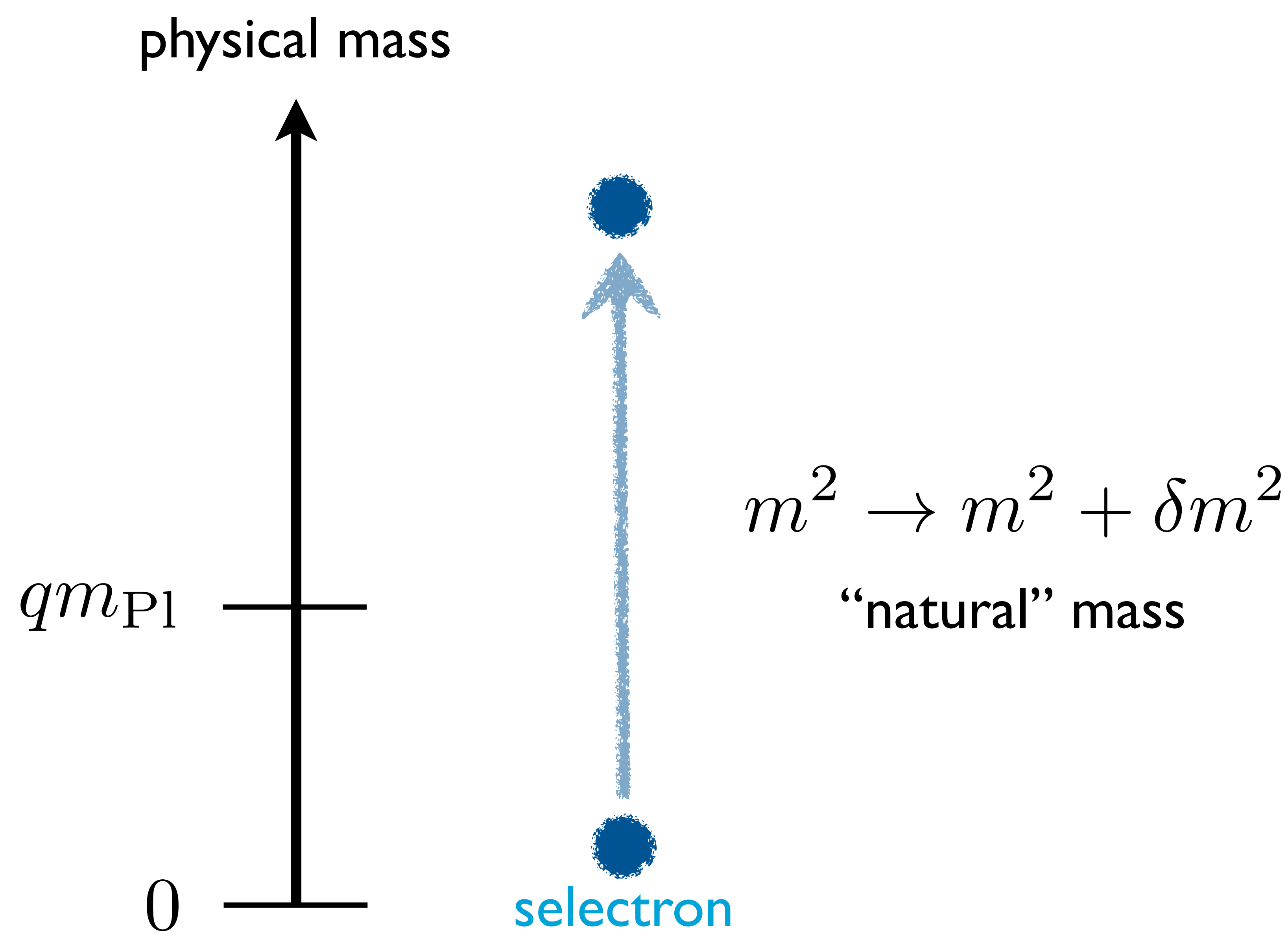

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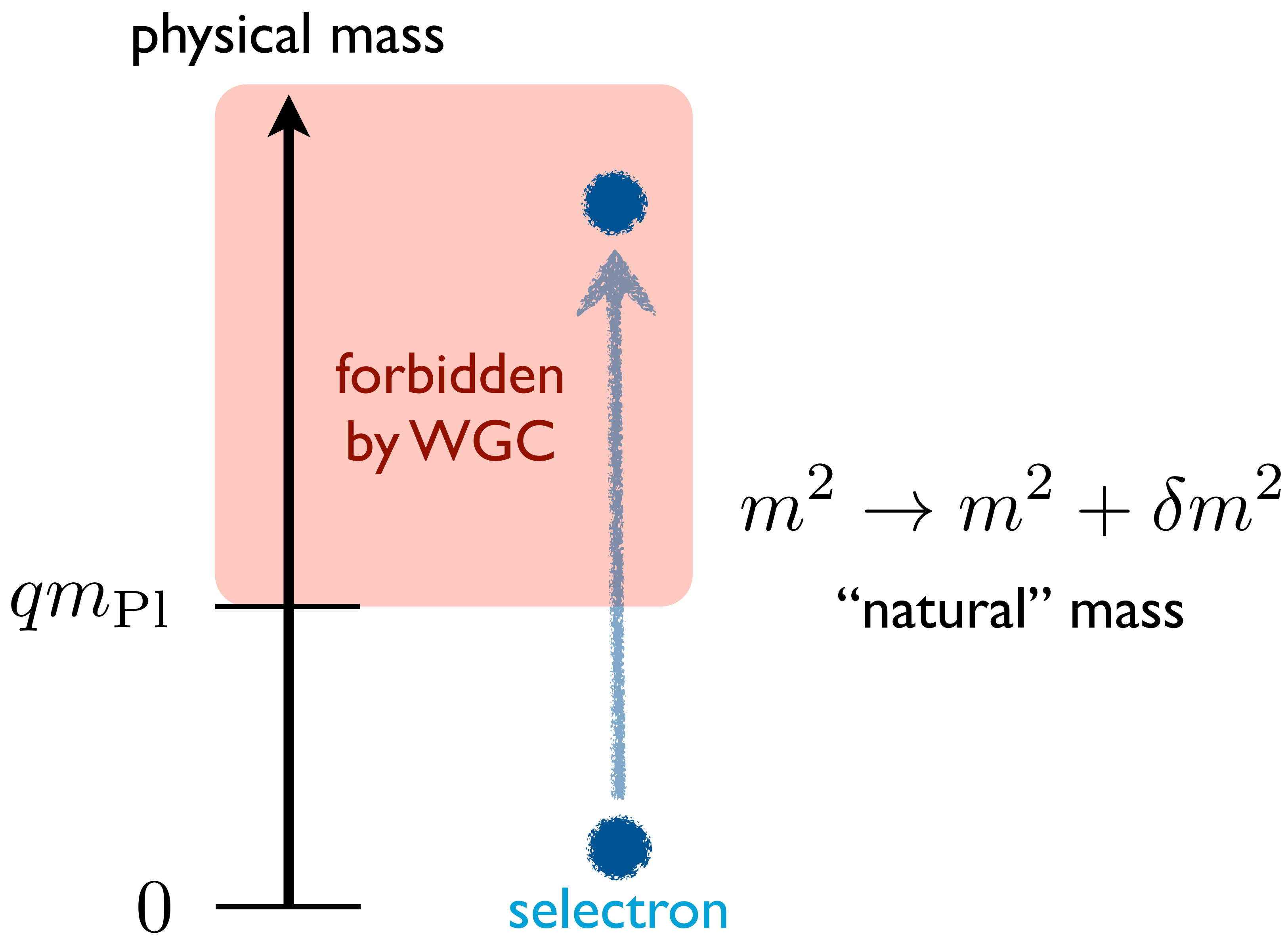

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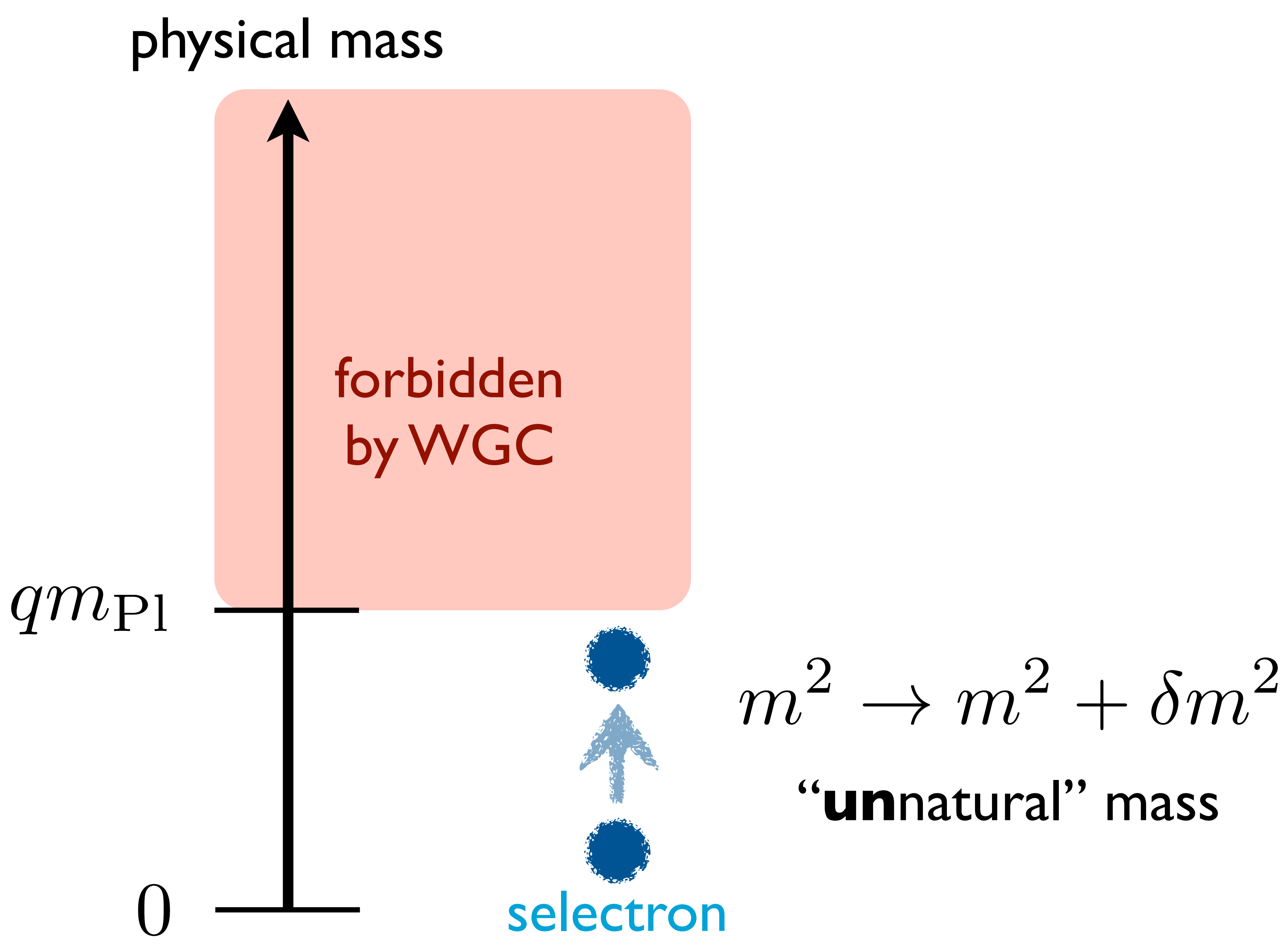

Let's quantify the tension.

$$m^2 \to m^2 + \delta m^2$$

$$\delta m^2 = \frac{\Lambda^2}{16\pi^2}(aq^2 + b\lambda)$$

incalculable coefficients

Naturalness principle: absent symmetries, the physical mass squared is $\sim \delta m^2$, so a, b are $\mathcal{O}(1)$ coefficients.

Setting the physical mass equal to its natural value yields a charge to mass ratio

$$z = q m_{\mathrm{Pl}}/m$$

$$= \frac{4\pi m_{\mathrm{Pl}}}{\Lambda} \frac{1}{\sqrt{a + b\lambda/q^2}}$$

Since the charged scalar is the only state in the spectrum, the WGC implies

$$z > 1$$

So, the loop cutoff is bounded from above.

$$\Lambda < \frac{4\pi m_{\mathrm{Pl}}}{\sqrt{a}} \qquad q^2 \gg \lambda$$

$$\Lambda < 4\pi m_{\mathrm{Pl}}\sqrt{\frac{q^2}{b\lambda}} \qquad q^2 \ll \lambda$$

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reasonable: cutoff below Planck

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(technically natural)

weird: cutoff parametrically below Planck!

lessons from scalar QED

- Some of the "natural" parameter space of scalar QED is in the swampland.

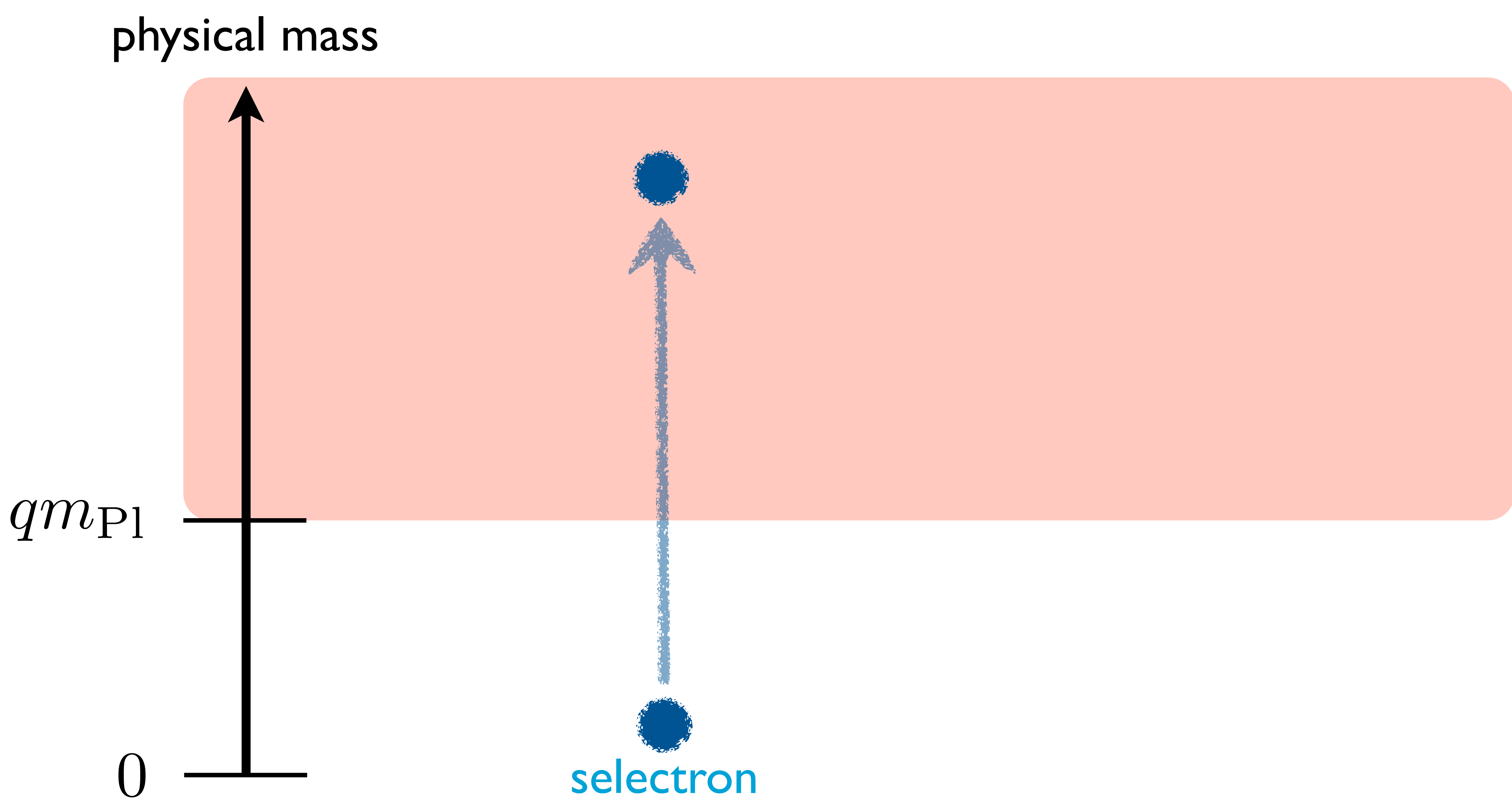
- More generally, naturalness and WGC can be mutually inconsistent in any theory with an Abelian force and fundamental scalars.

naturalness $\longrightarrow \overset{?}{} \longleftarrow$ consistency with quantum gravity

Naturalness and WGC *can* be reconciled if we revisit and *modify* our premises.

There are three obvious strategies i), ii), iii).

i) Add new degrees of freedom below cutoff.



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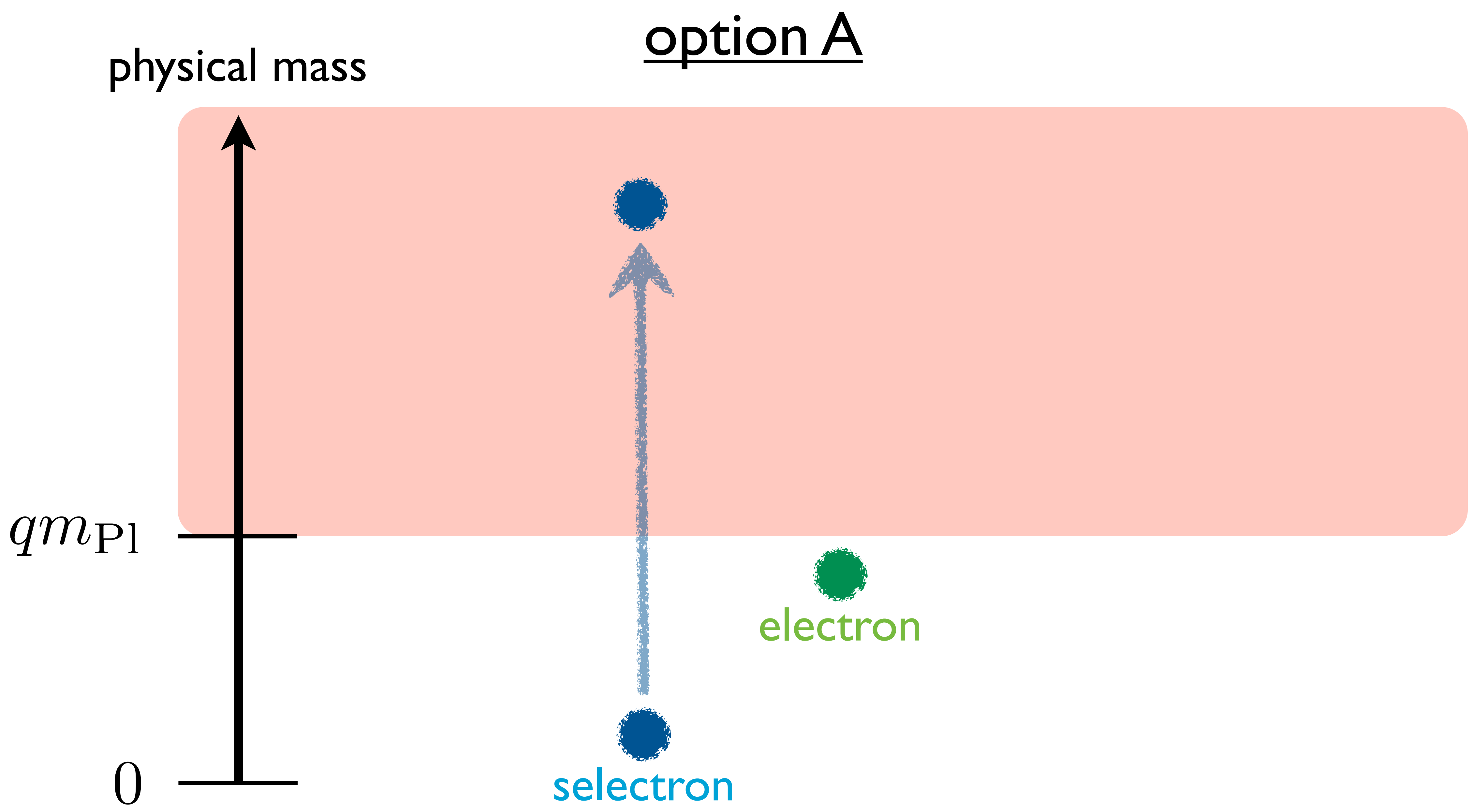

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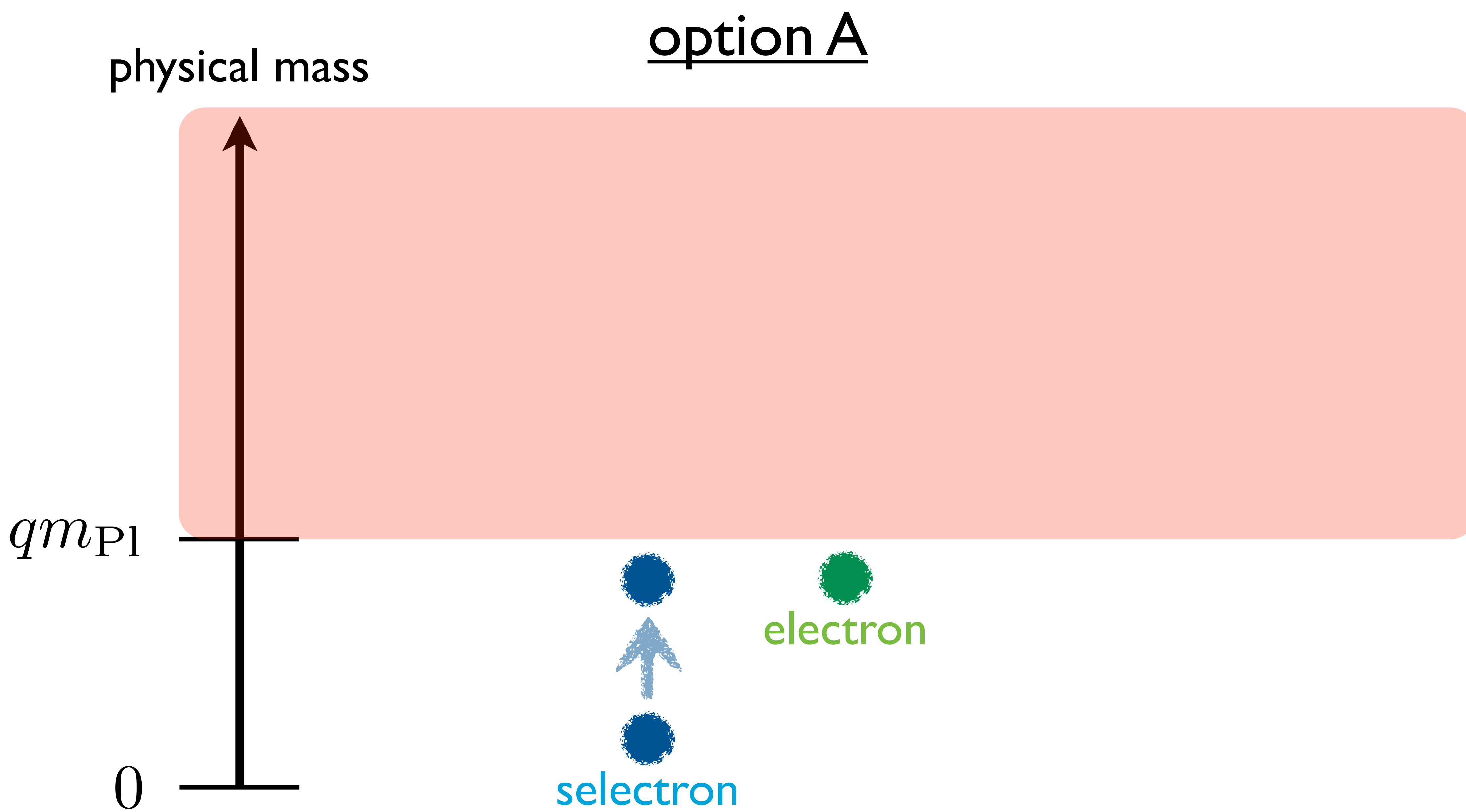

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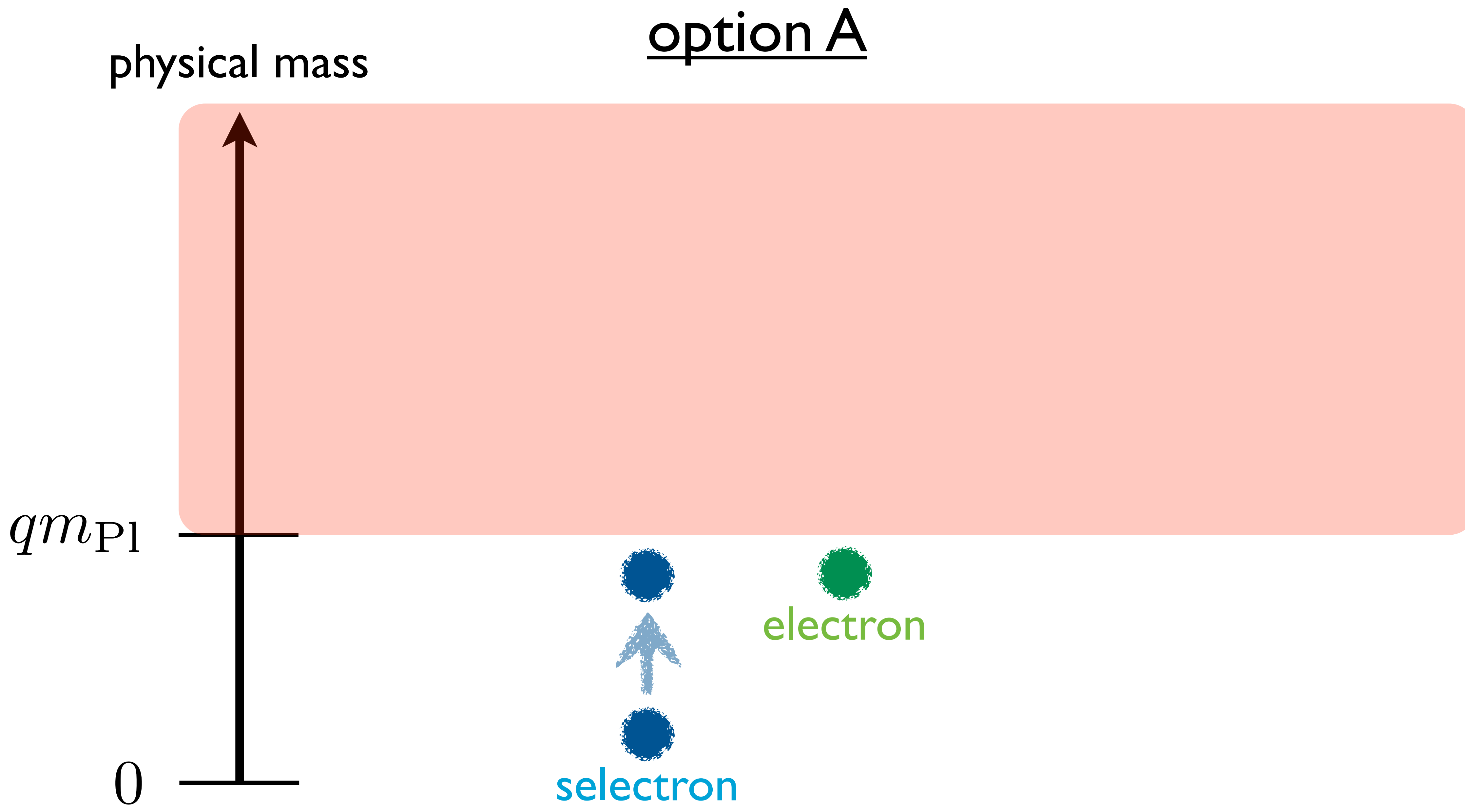

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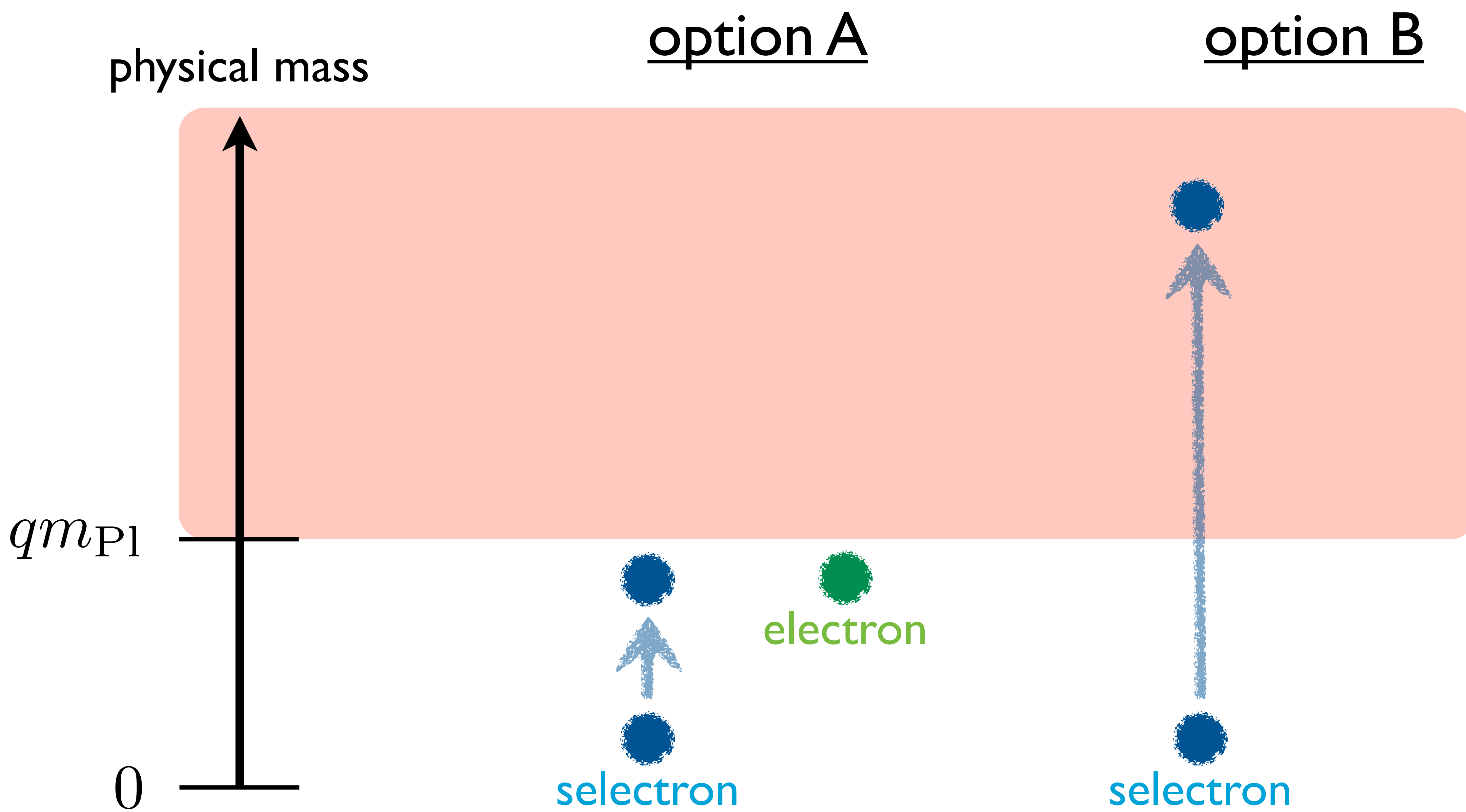

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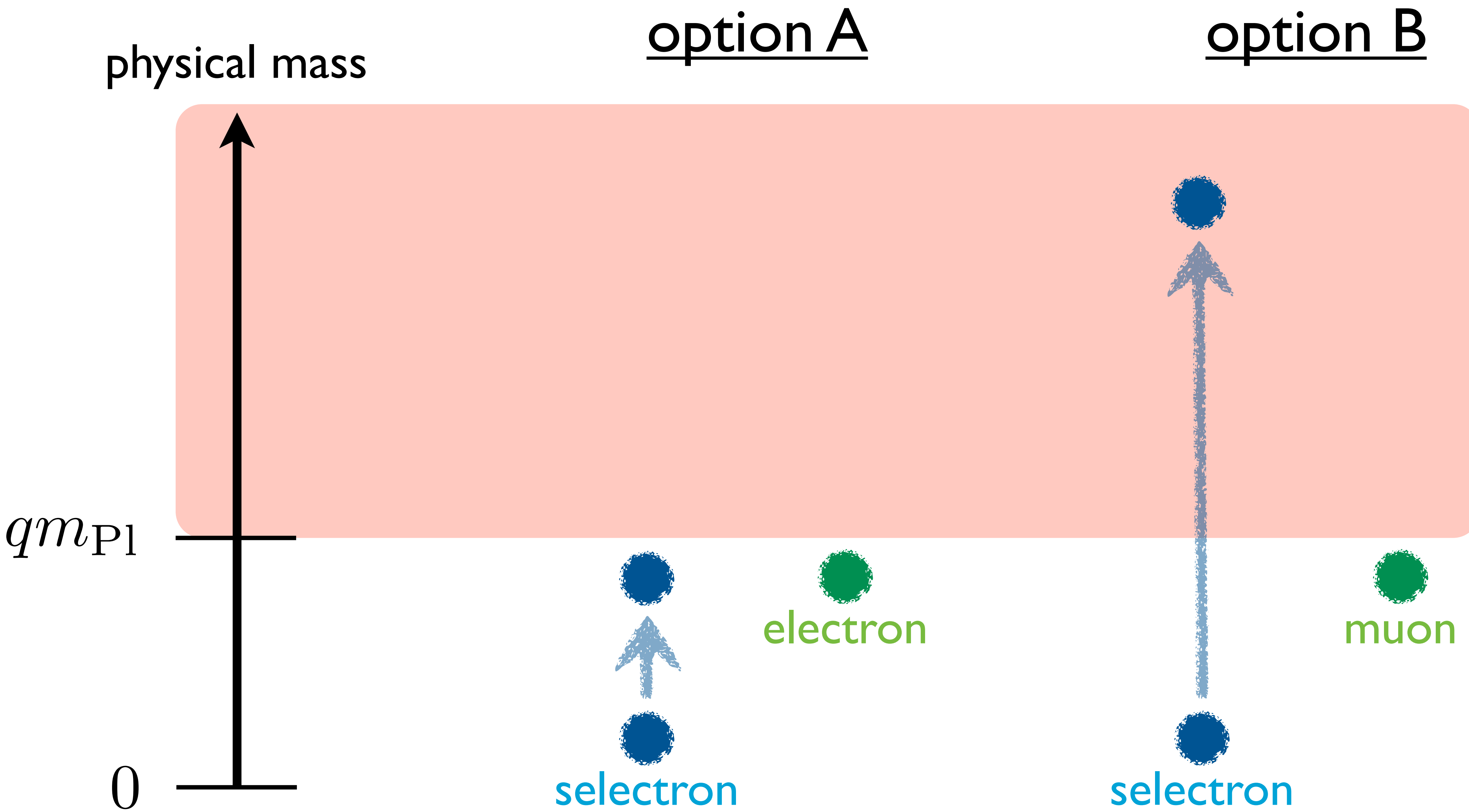

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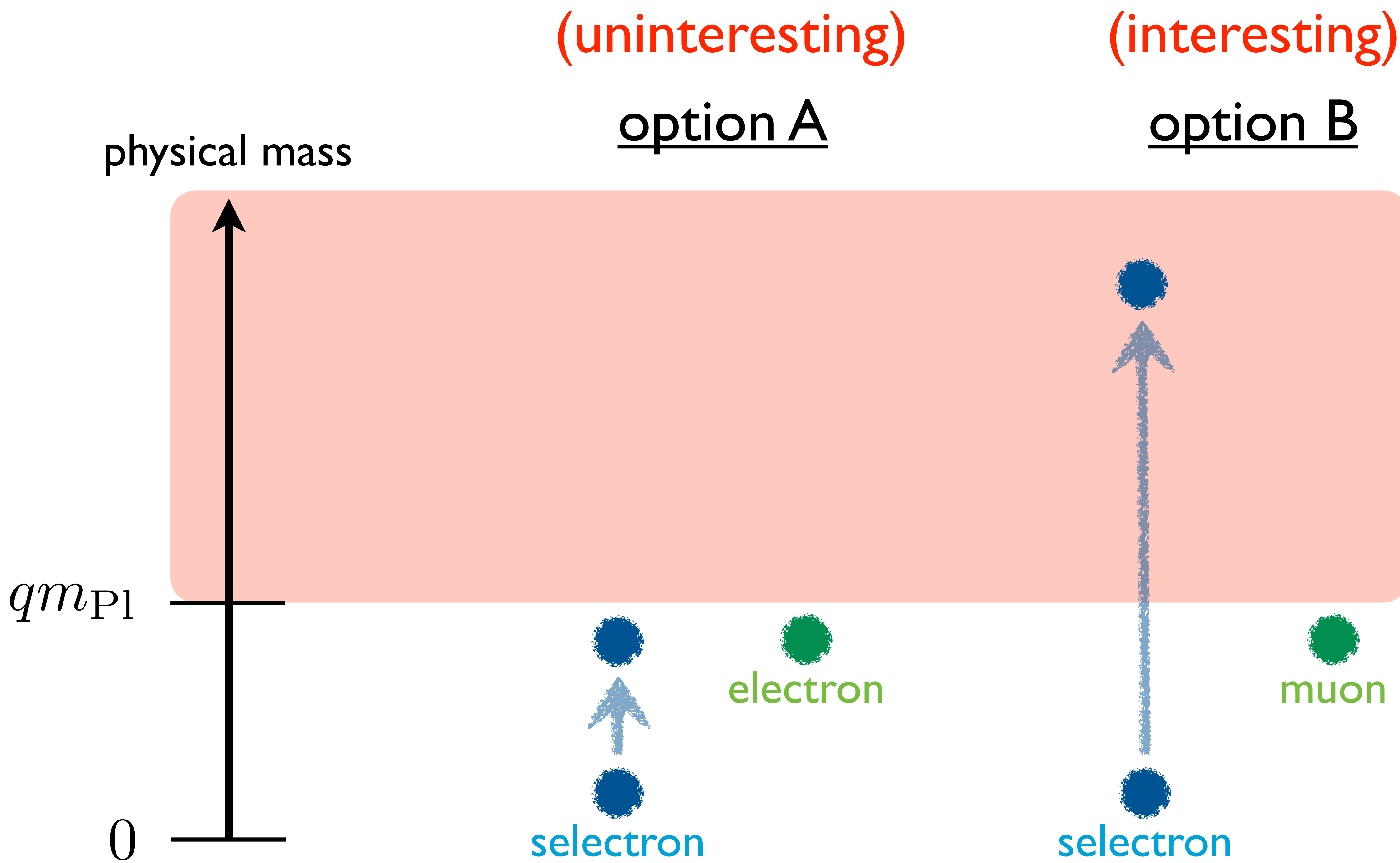

ii) Hierarchical couplings are forbidden.

$$q^2 \cancel{\ll} \lambda \longrightarrow q^2 \sim \lambda$$ (e.g. SUSY D-terms)

Of course, something more than SUSY must be required to justify this possibility.

Still, WGC implies that "little hierarchical" couplings will impose "little hierarchies".

iii) Theory is driven to a Higgs phase.

$$\delta m^2 < 0$$

The WGC is ambiguous in the Higgs phase because $[q, m] \neq 0$. Whose mass, charge?

More importantly, black holes do not have Higgsed U(1) hair. No justification for WGC!

Ideally, we'd like to connect WGC to our universe, which exhibits *multiple* forces and *multi-charged* states.

What is the generalization of the WGC?

generalized WGC

For a U(1)^N theory, photons form an SO(N) multiplet. Define SO(N) charge vectors:

$$q_{ia} = \vec{q}_i$$

species index photon index

Also define SO(N) charge to mass vectors:

$$\vec{z}_i = \vec{q}_i \, m_{\mathrm{Pl}}/m_i$$

We can try to guess the generalized WGC.
The condition should be SO(N) invariant.

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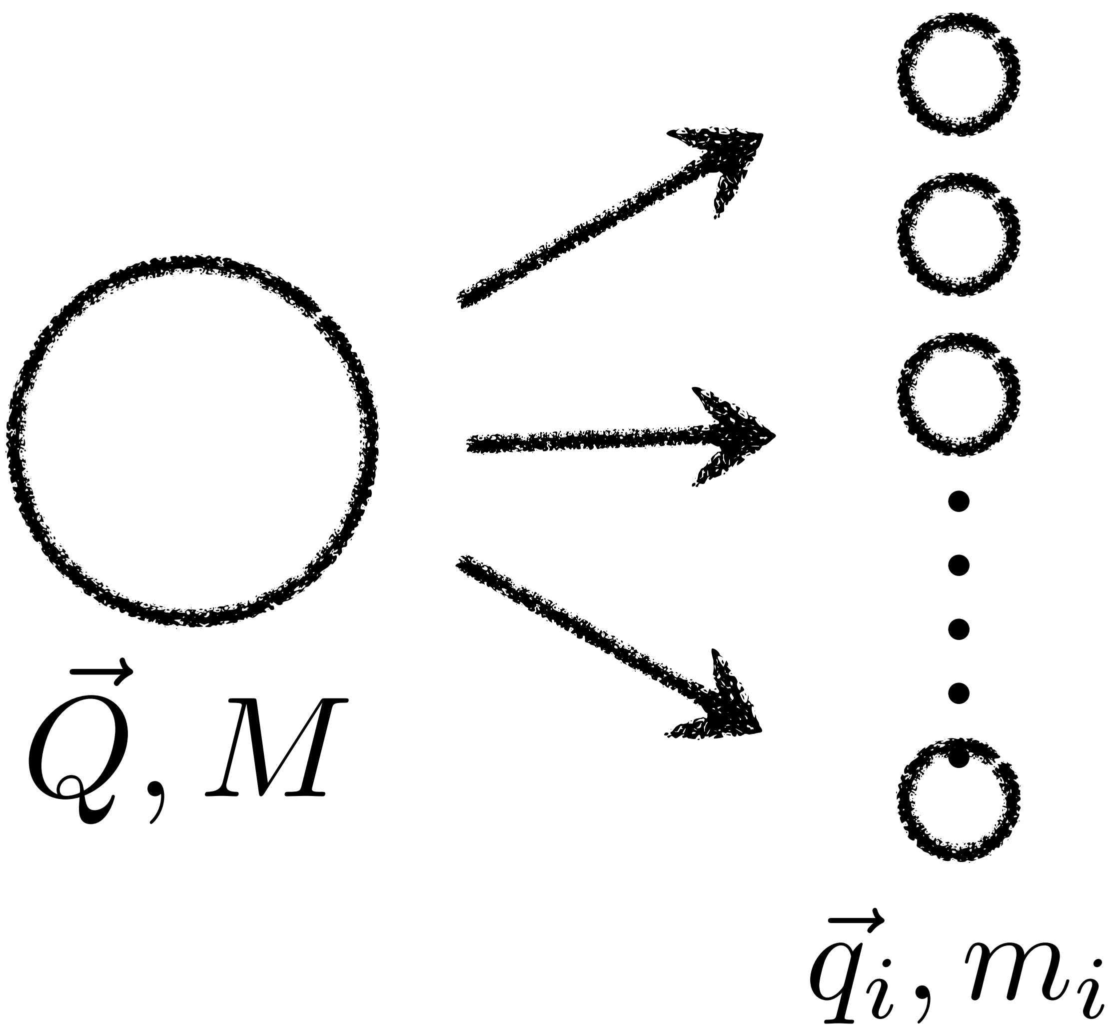
(still not sufficient!)

Let's just derive the generalized WGC.



charge conservation

$$\vec{Q} = \sum_i n_i \vec{q}_i$$

energy conservation

$$M > \sum_i n_i m_i$$

Black hole decays to n_i particles of species i.

Going to charge to mass ratio variables:

$$\vec{Q} = \sum_i n_i \vec{q}_i \qquad M > \sum_i n_i m_i$$

$$\downarrow \qquad\qquad \downarrow$$

$$\vec{Z} = \sum_i \sigma_i \vec{z}_i \qquad 1 > \sum_i \sigma_i$$

where $\sigma_i = n_i m_i / M$ is the fractional mass.

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convex hull
spanned by vectors

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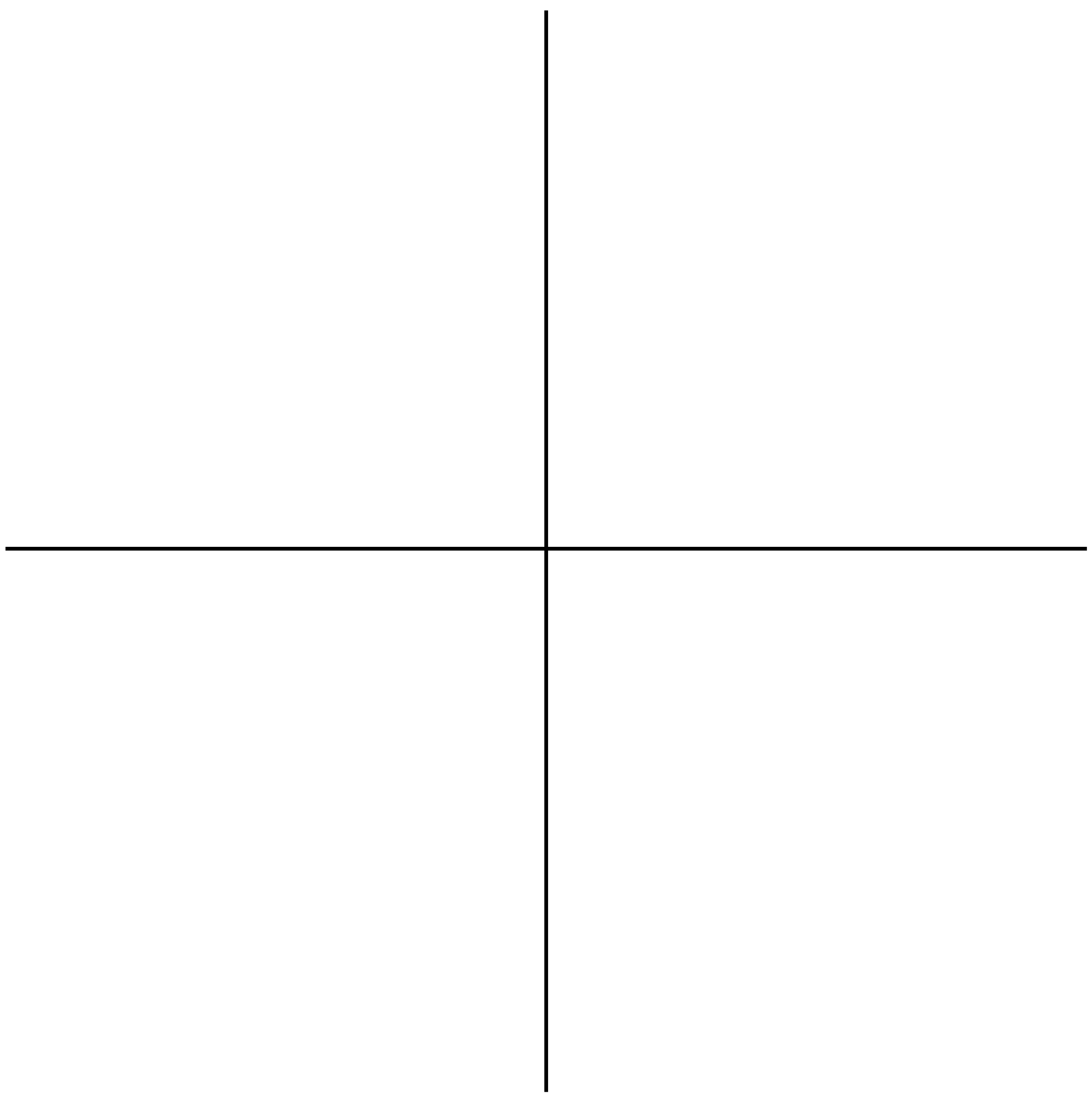

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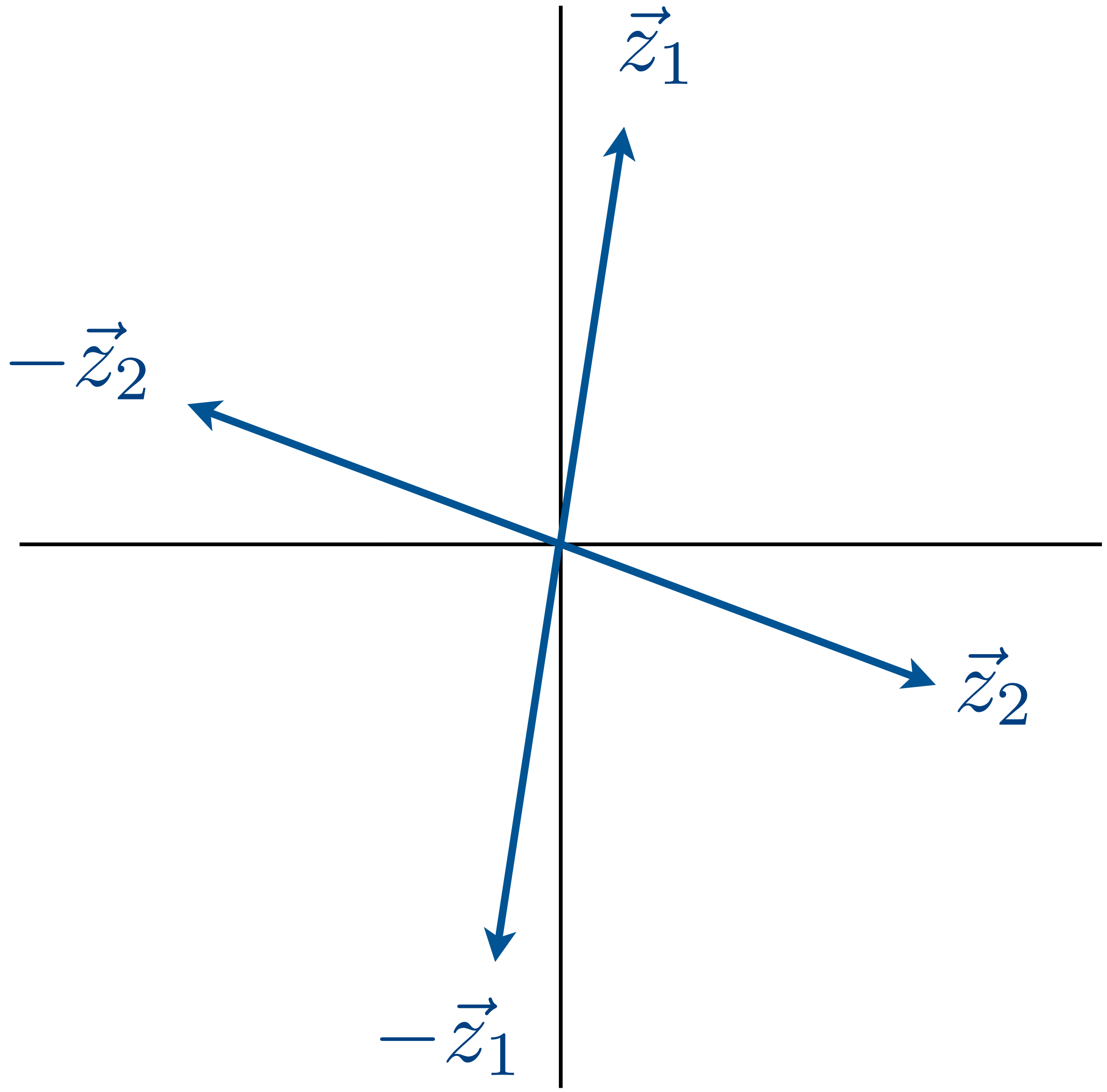

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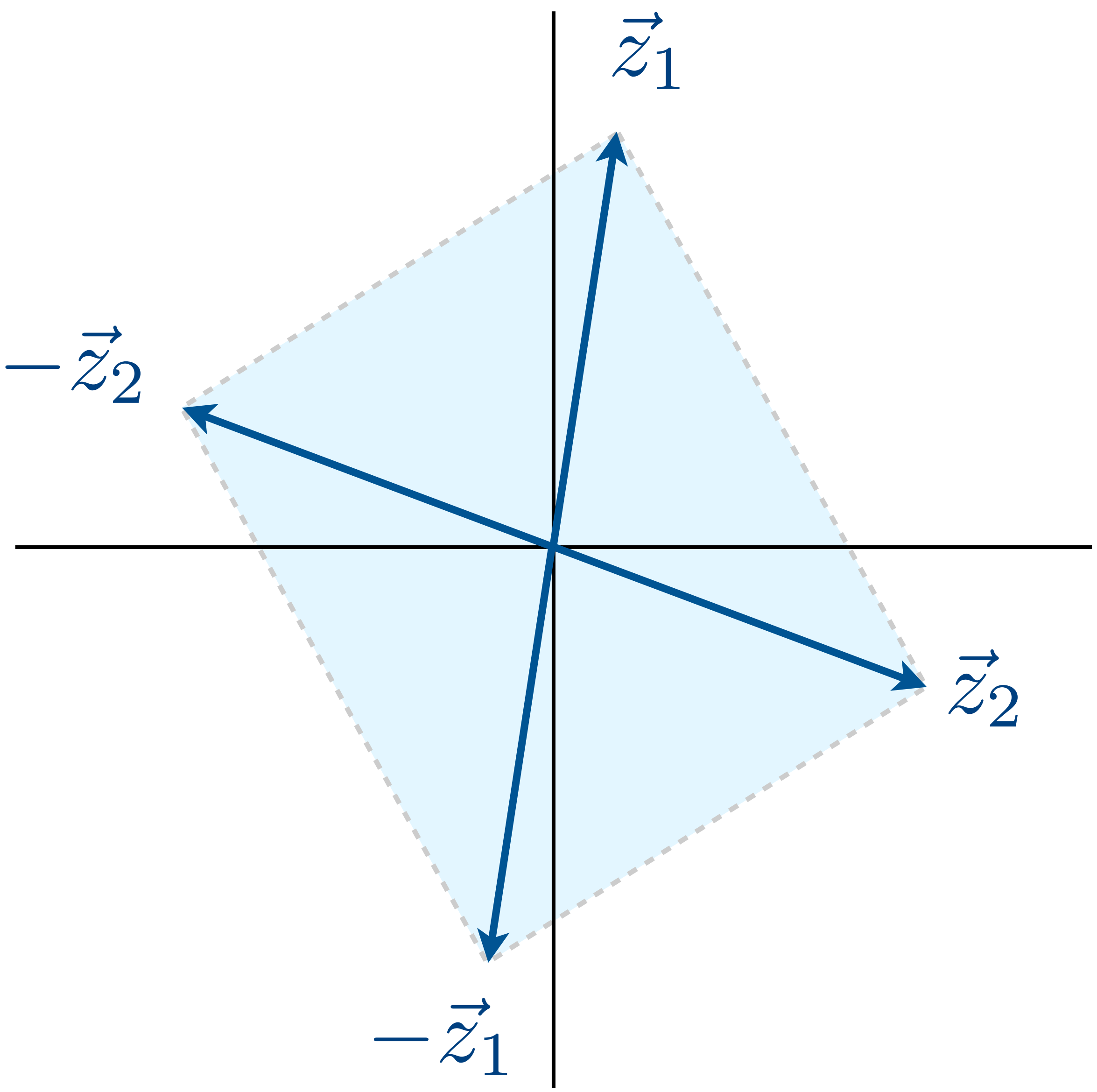

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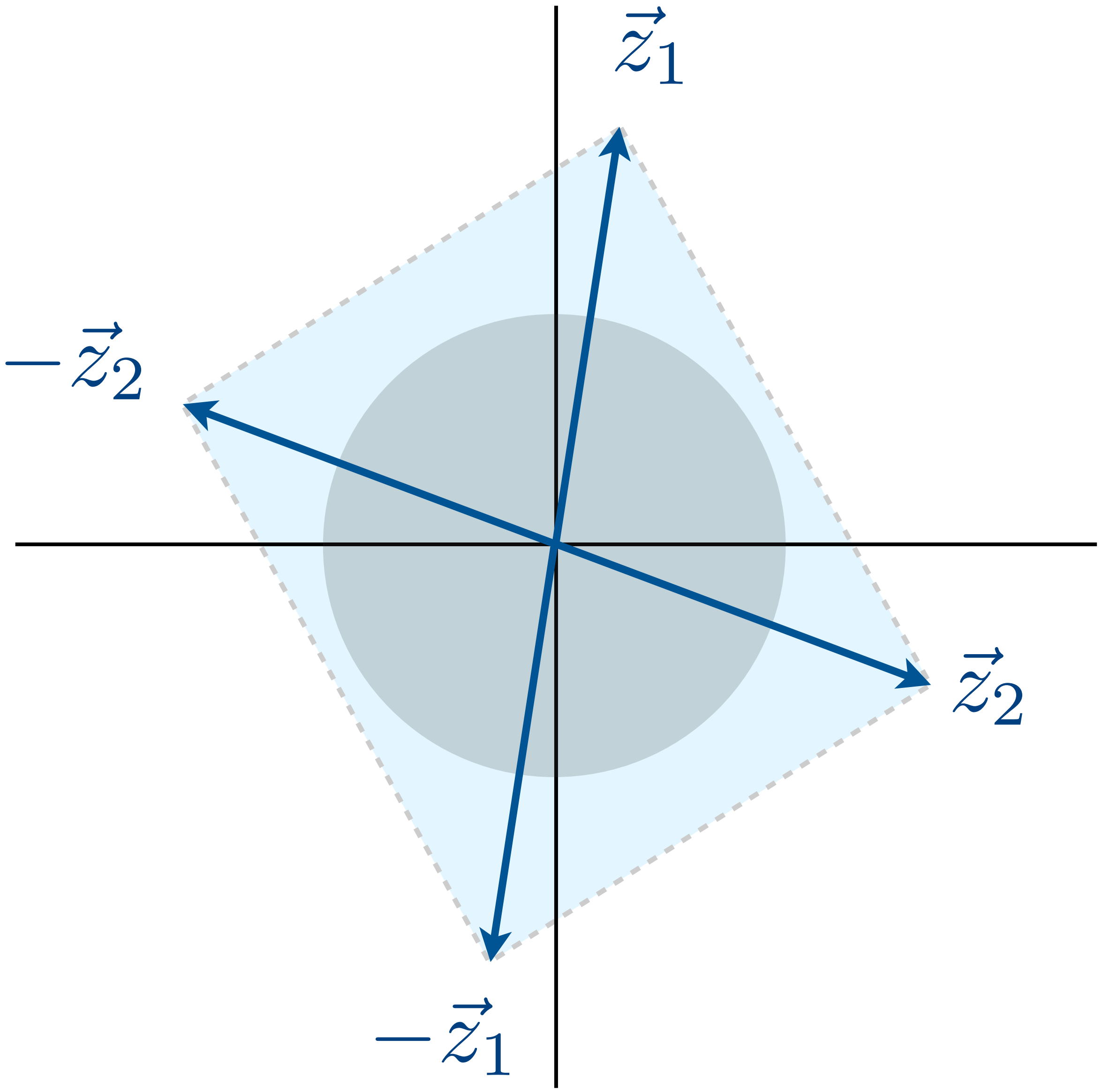

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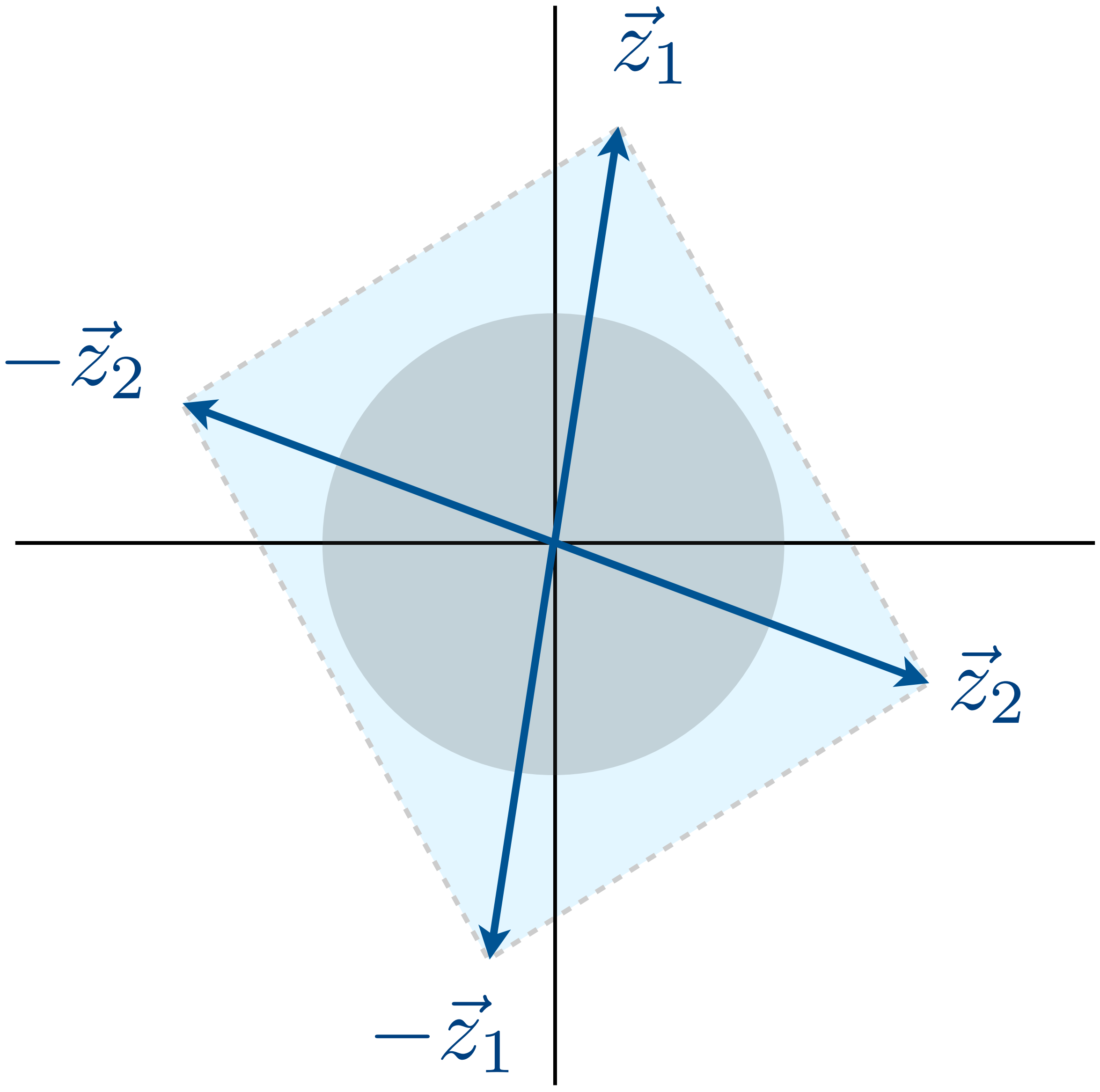
consistent with WGC



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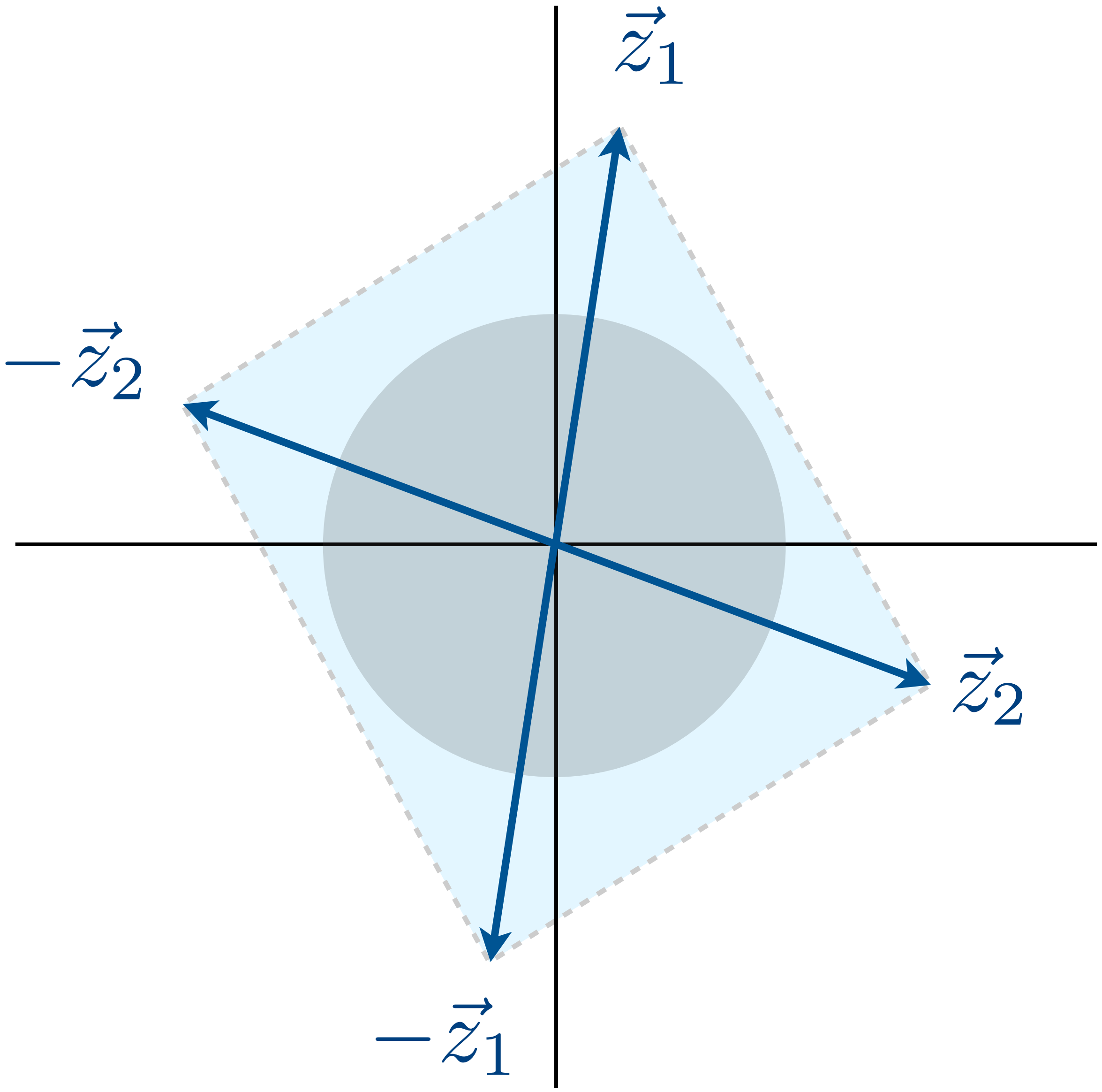

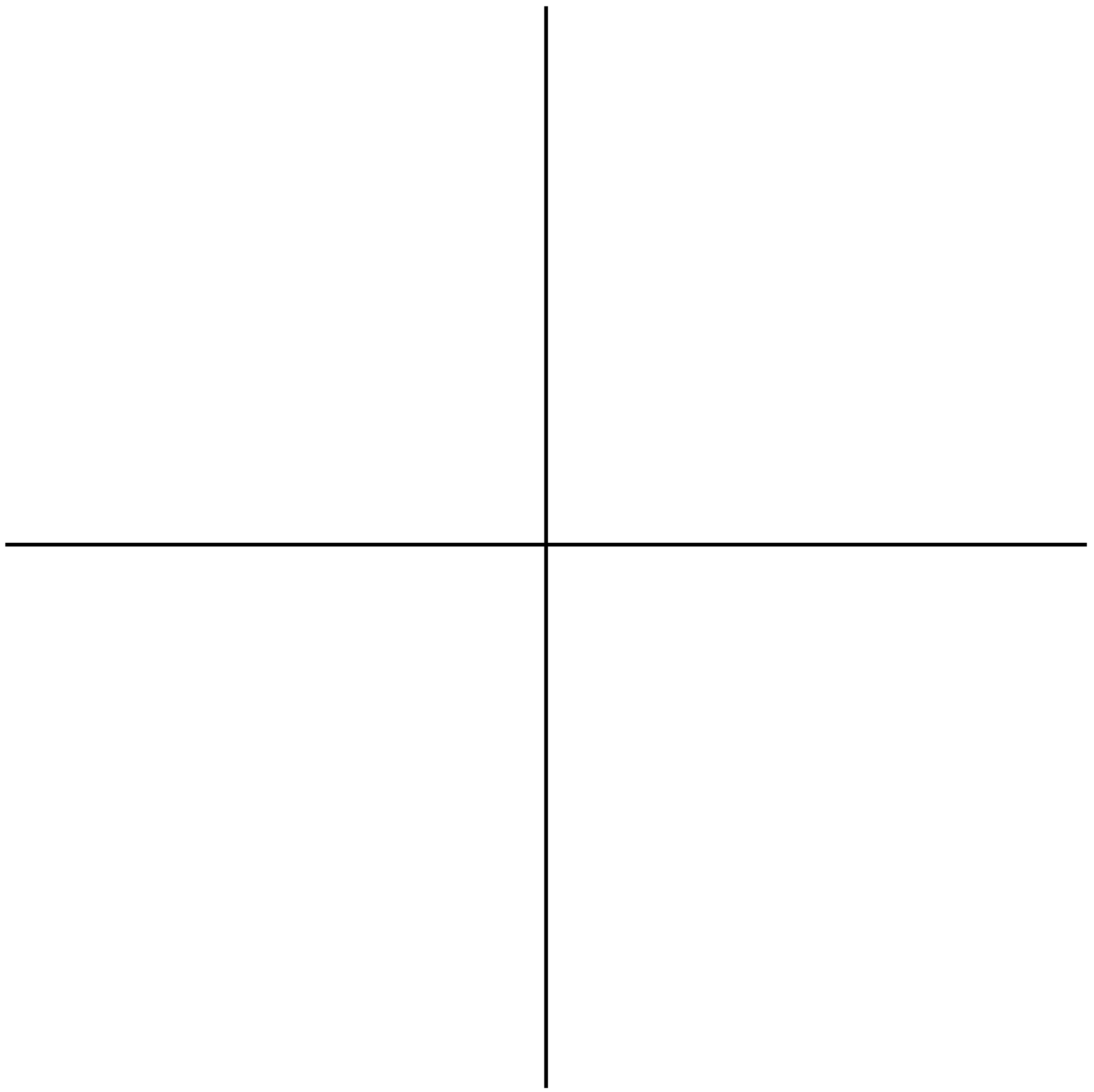

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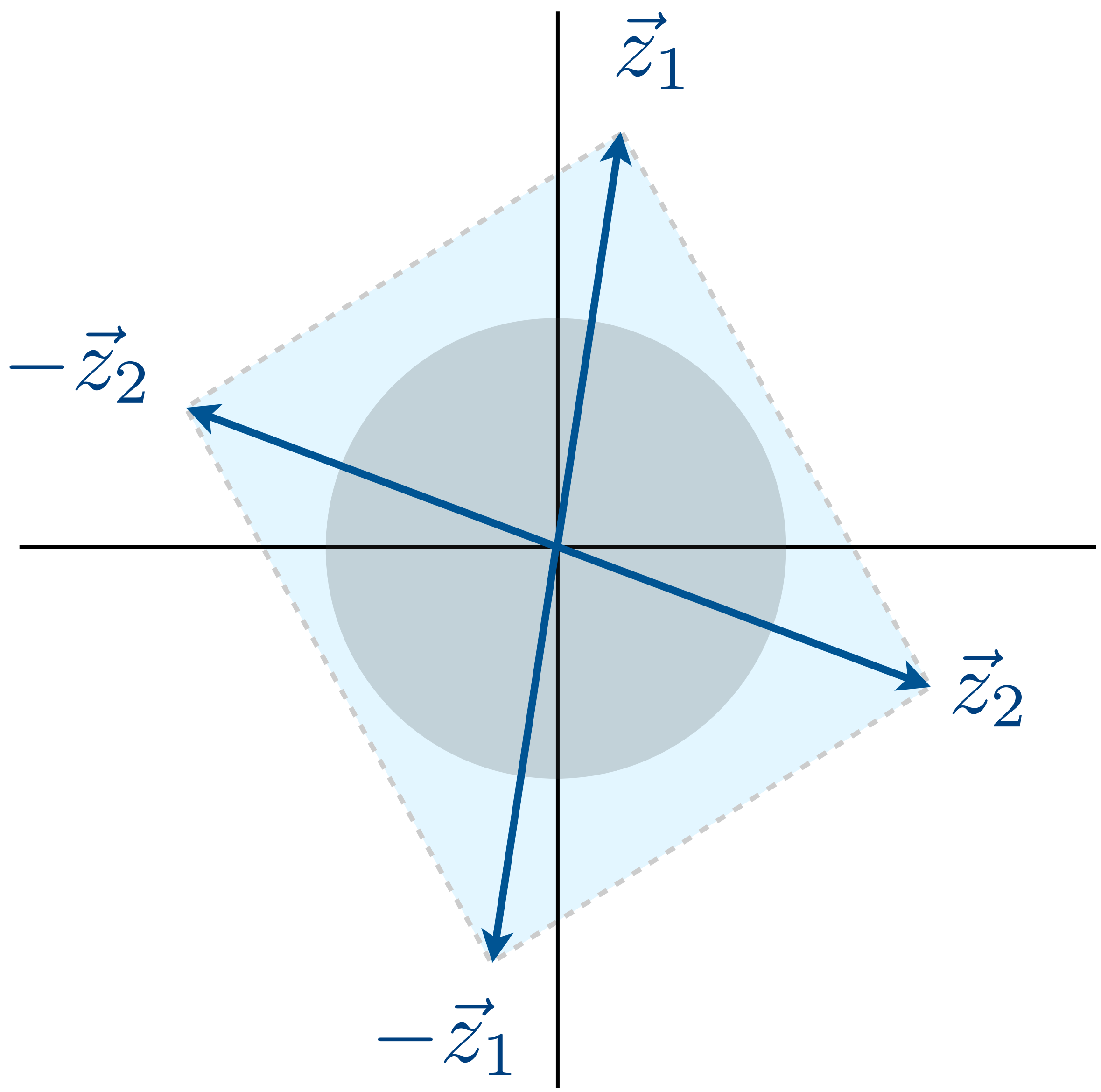

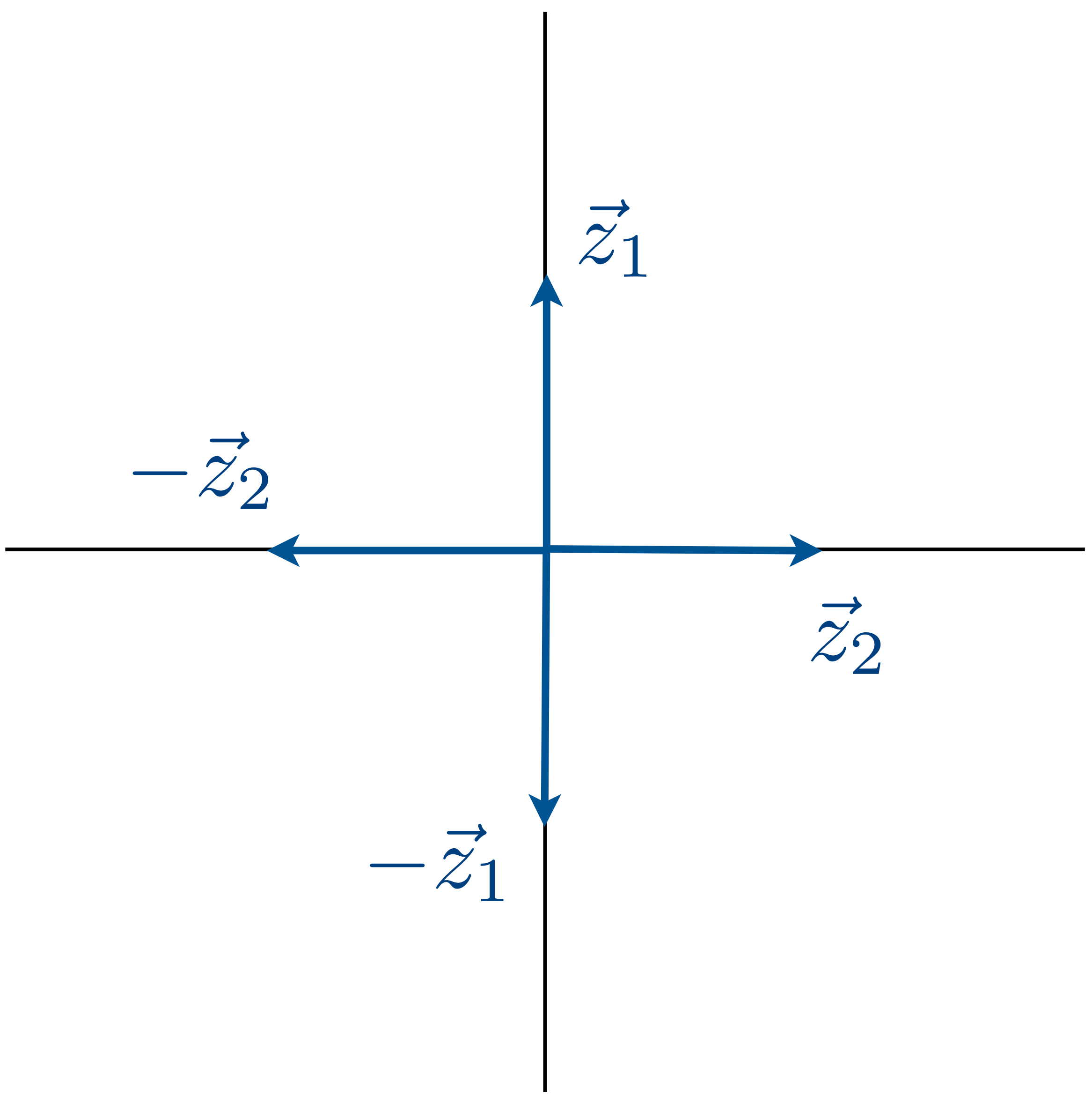

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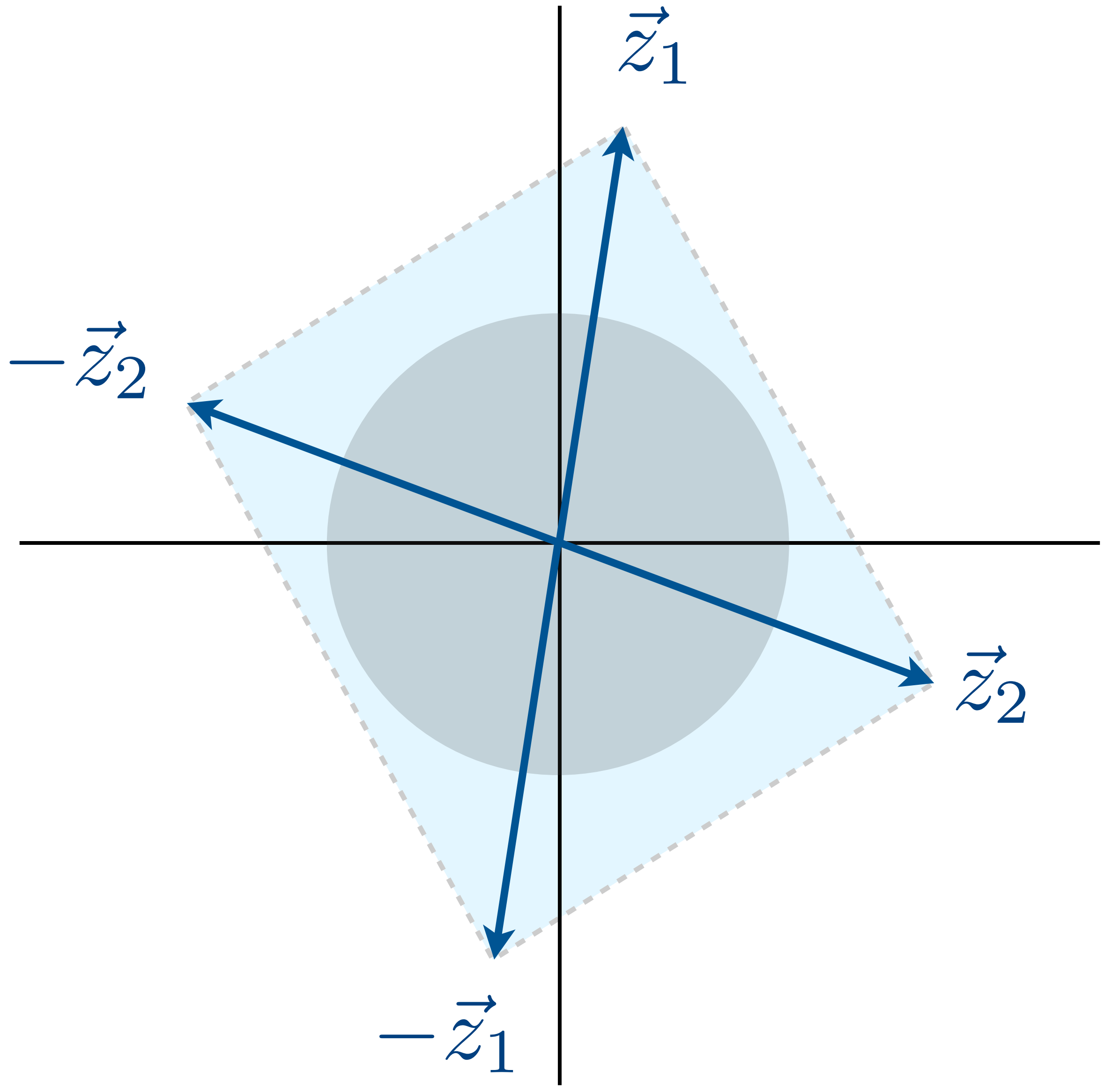

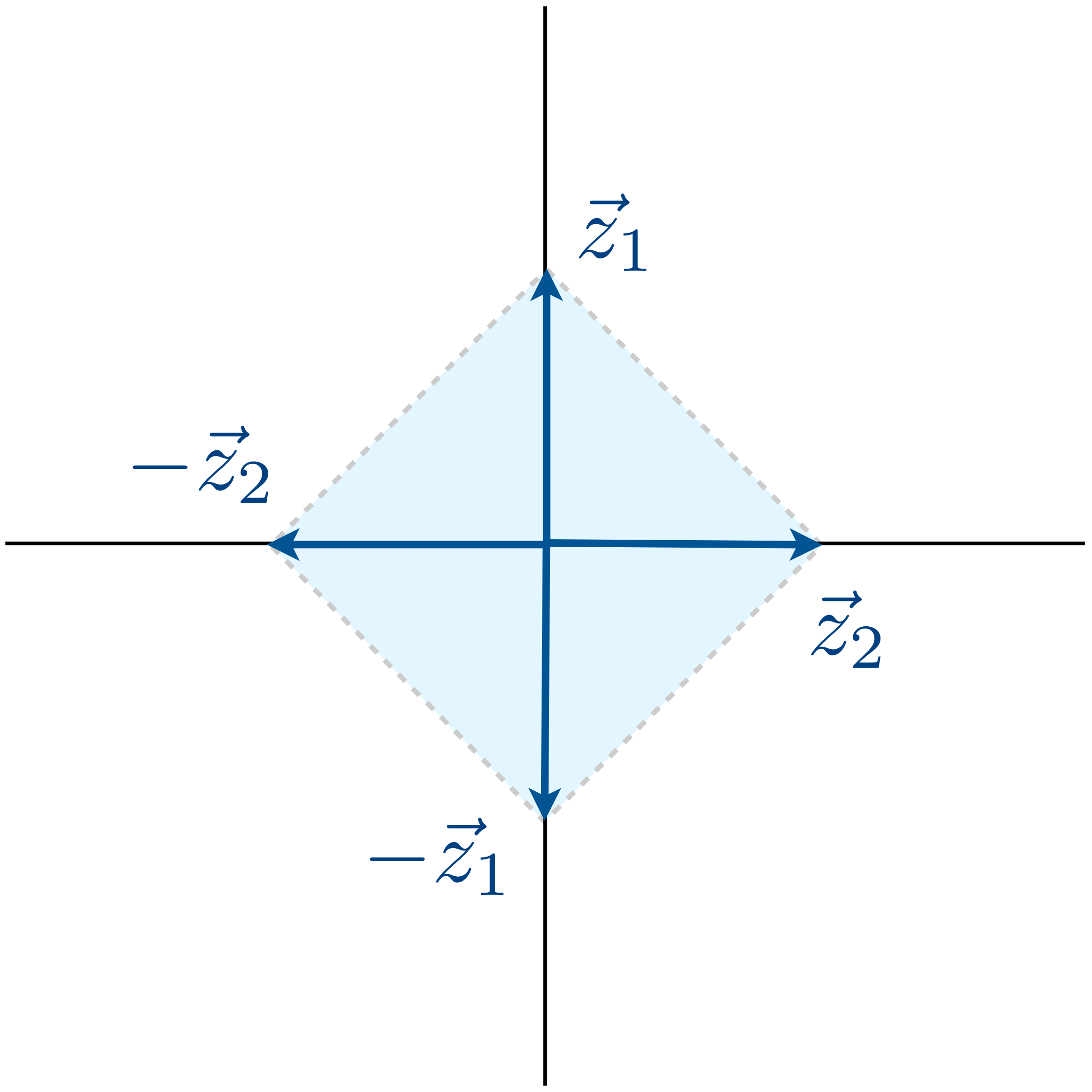

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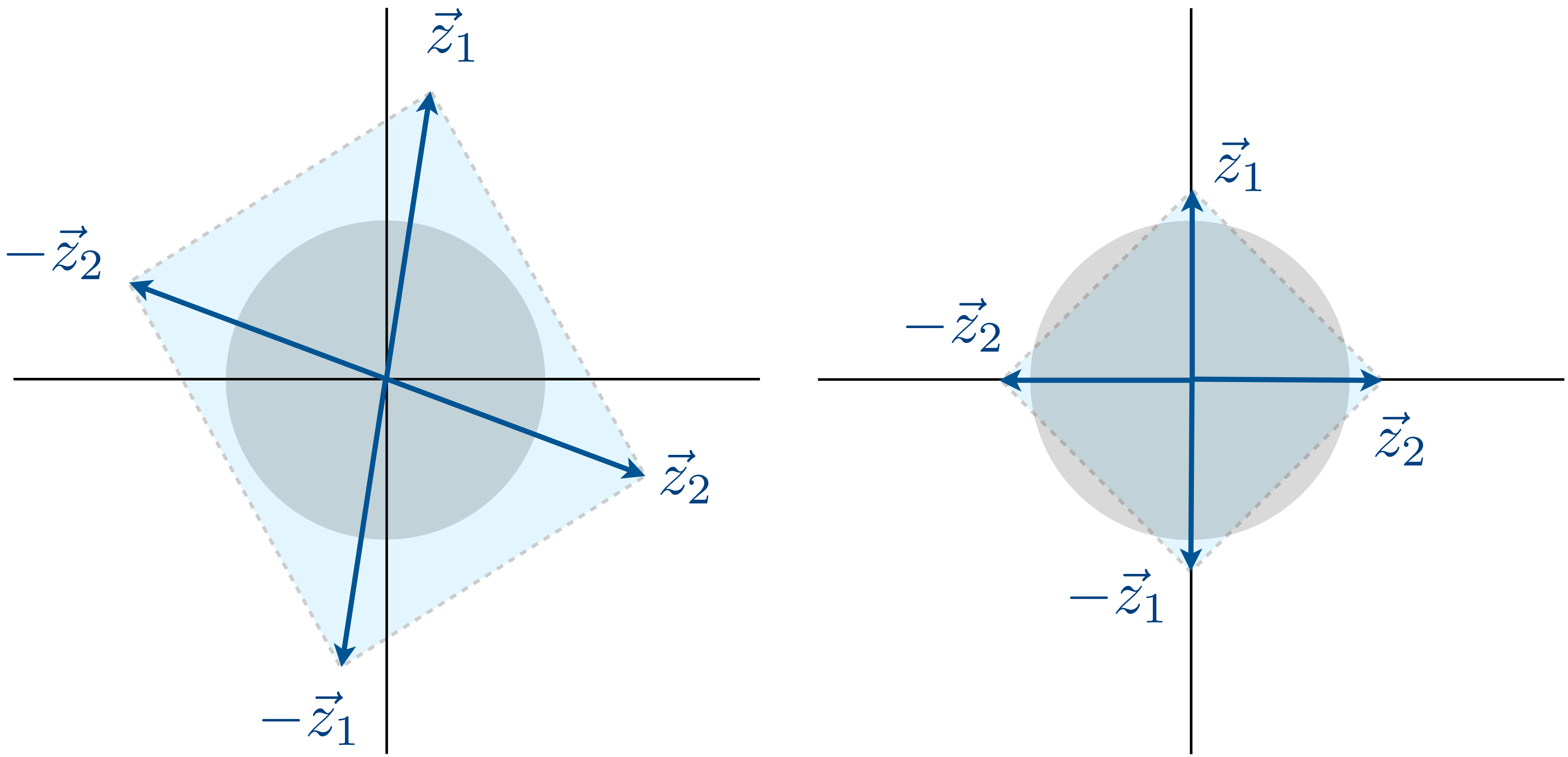

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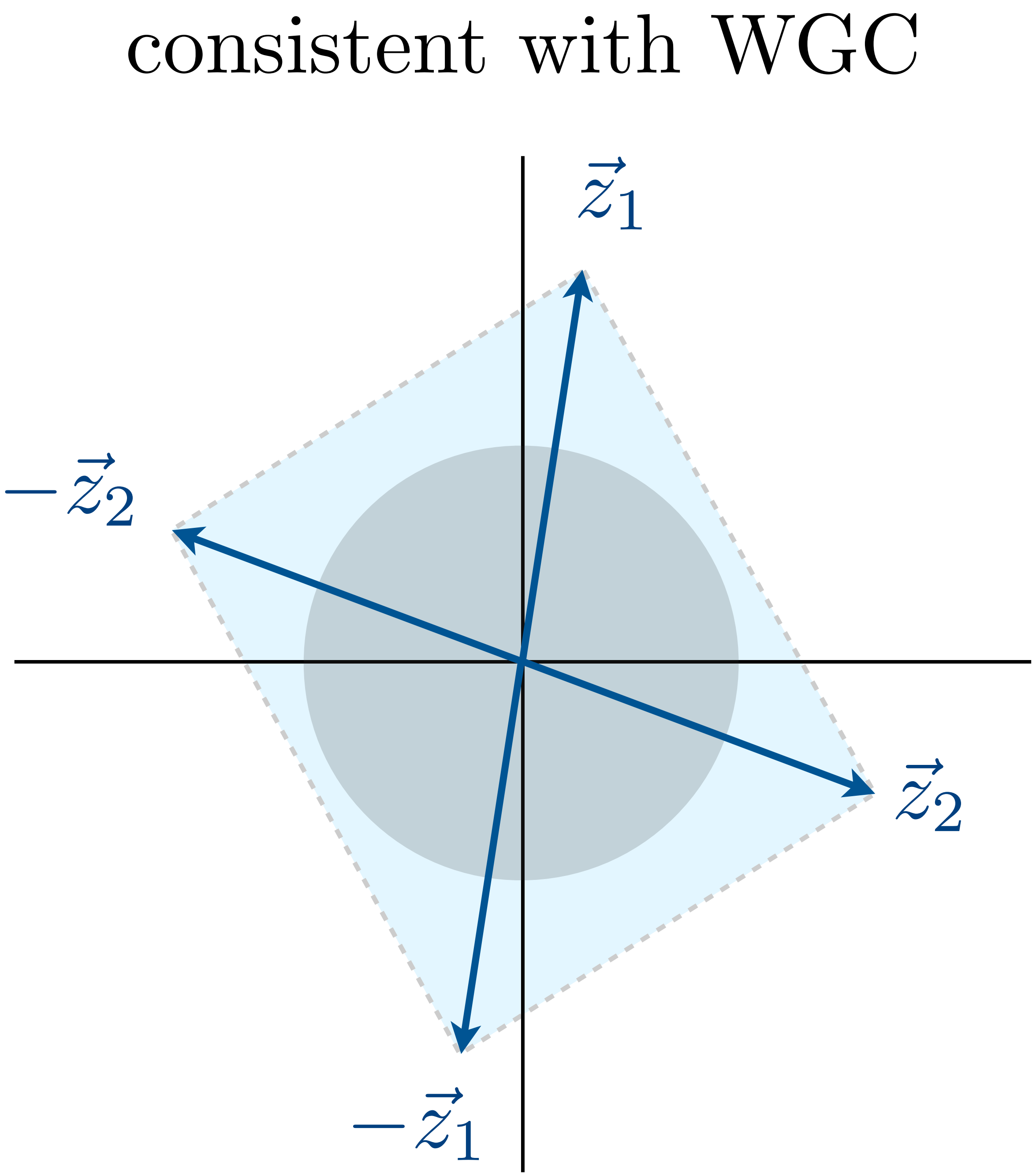

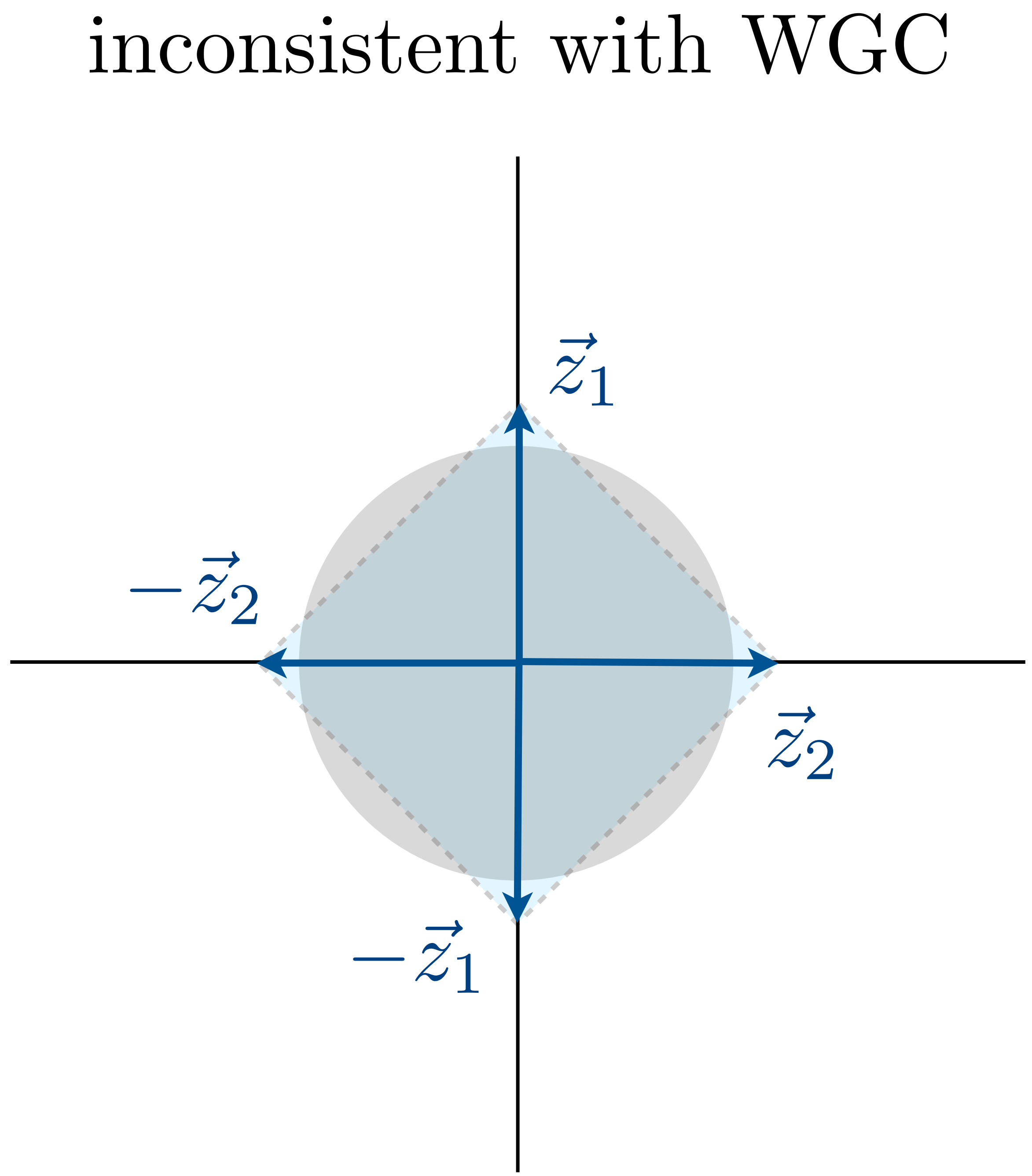

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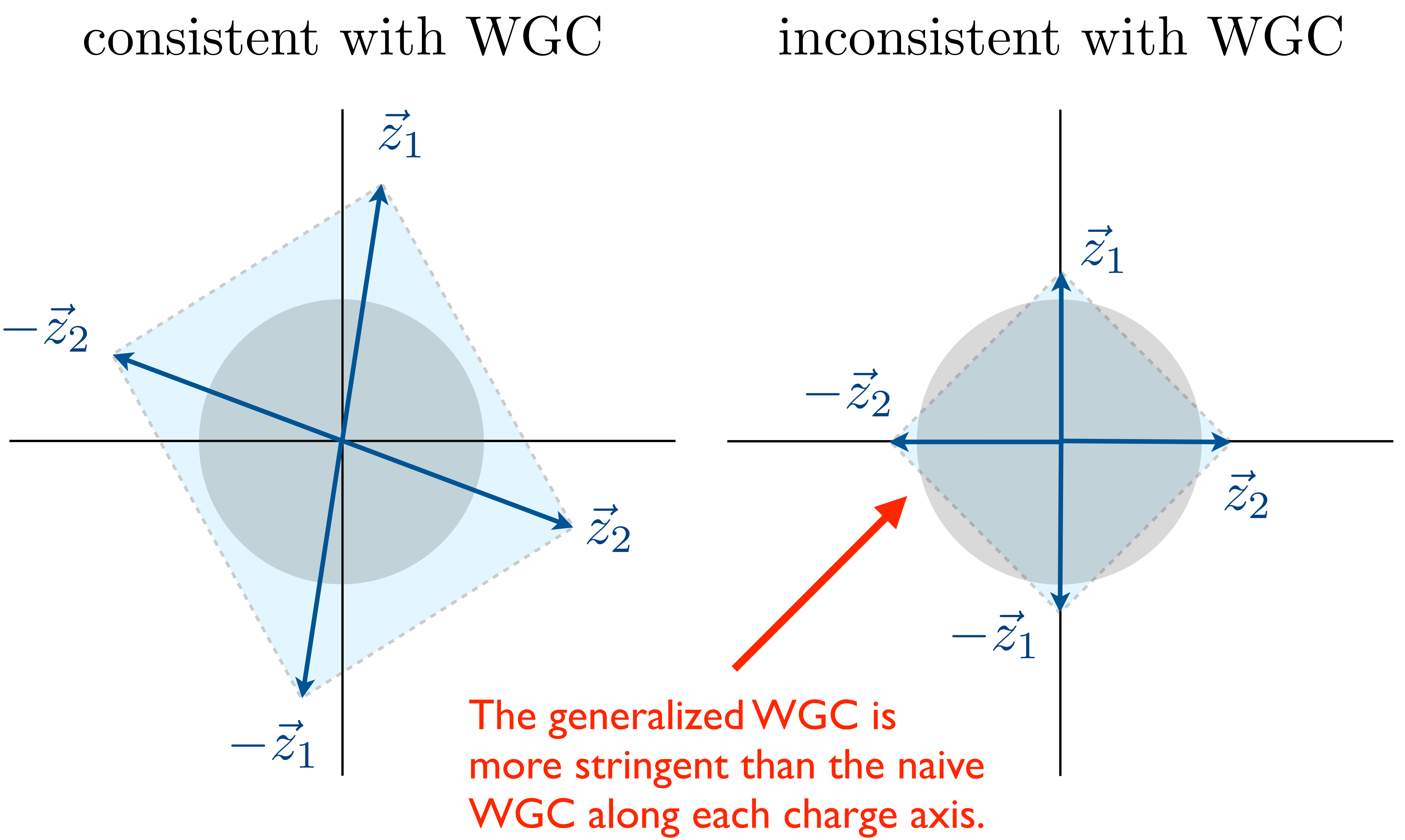


generalized WGC

- Draw convex hull spanned by each particle and anti-particle species, $\pm\vec{z}_i$.

- Draw the unit ball, $|\vec{Z}| = 1$, corresponding to extremal black holes.

- If the unit ball is contained in the convex hull, then WGC is satisfied.

Lastly, let's consider possible implications for physics beyond the standard model.

implications for physics beyond the standard model

At the very least, be more suspicious of "little hierarchy" problems.

$$q \sim 10^{-3}$$

$$\lambda \sim 1$$

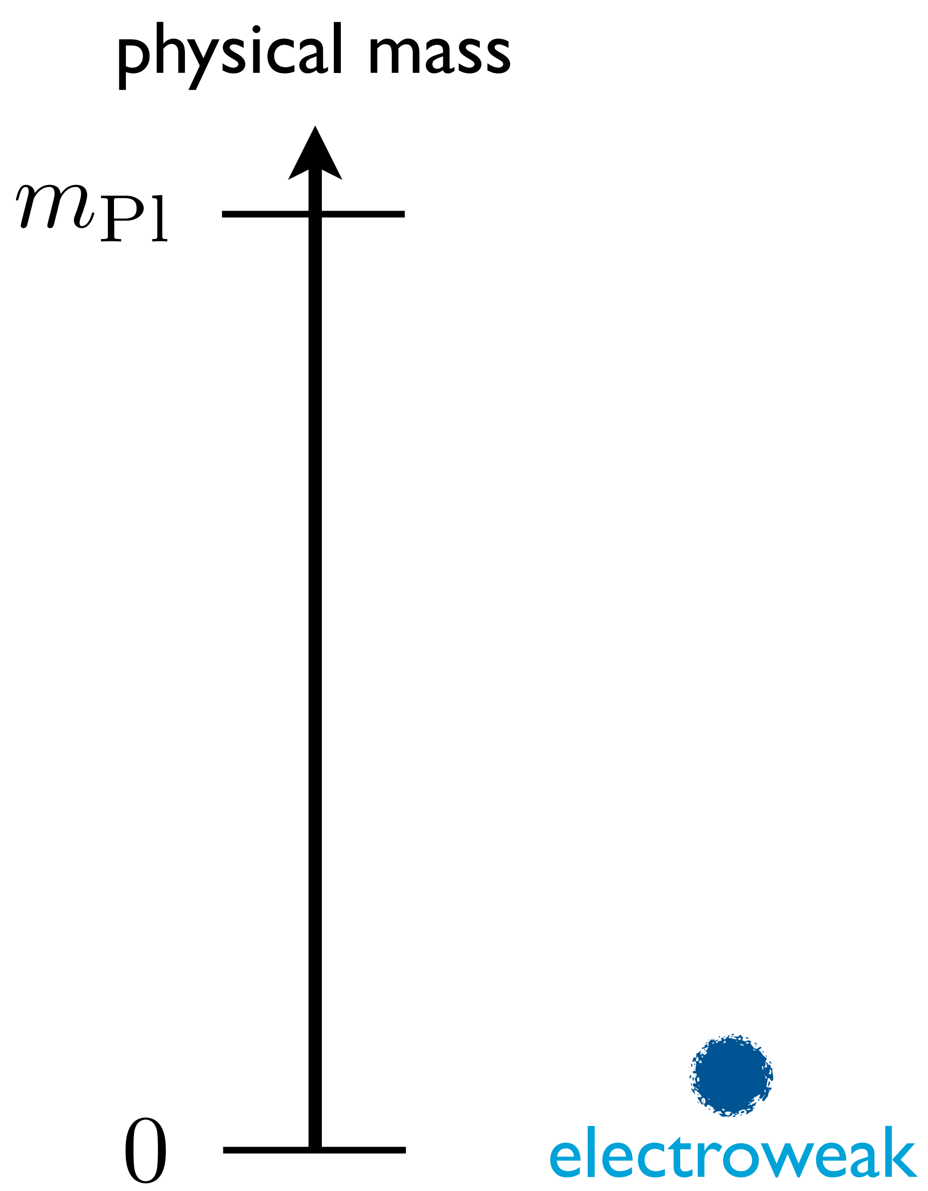
$\longrightarrow$

(little hierarchy)

$$\frac{q}{\sqrt{\lambda}} \sim 10^{-3}$$

Ultraviolet consistency may enforce small mass parameters in the infrared!

What about the "big" hierarchy problem?



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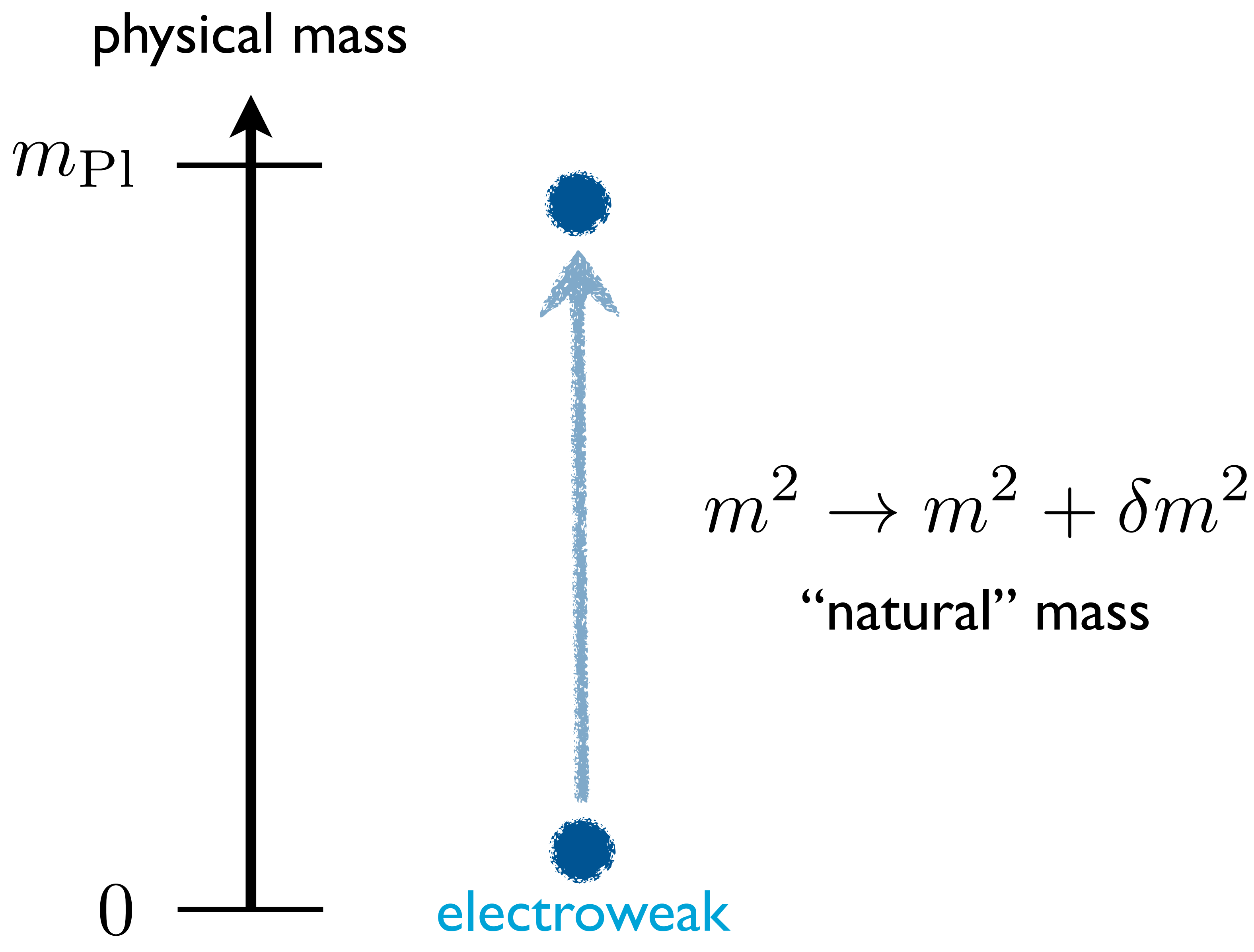

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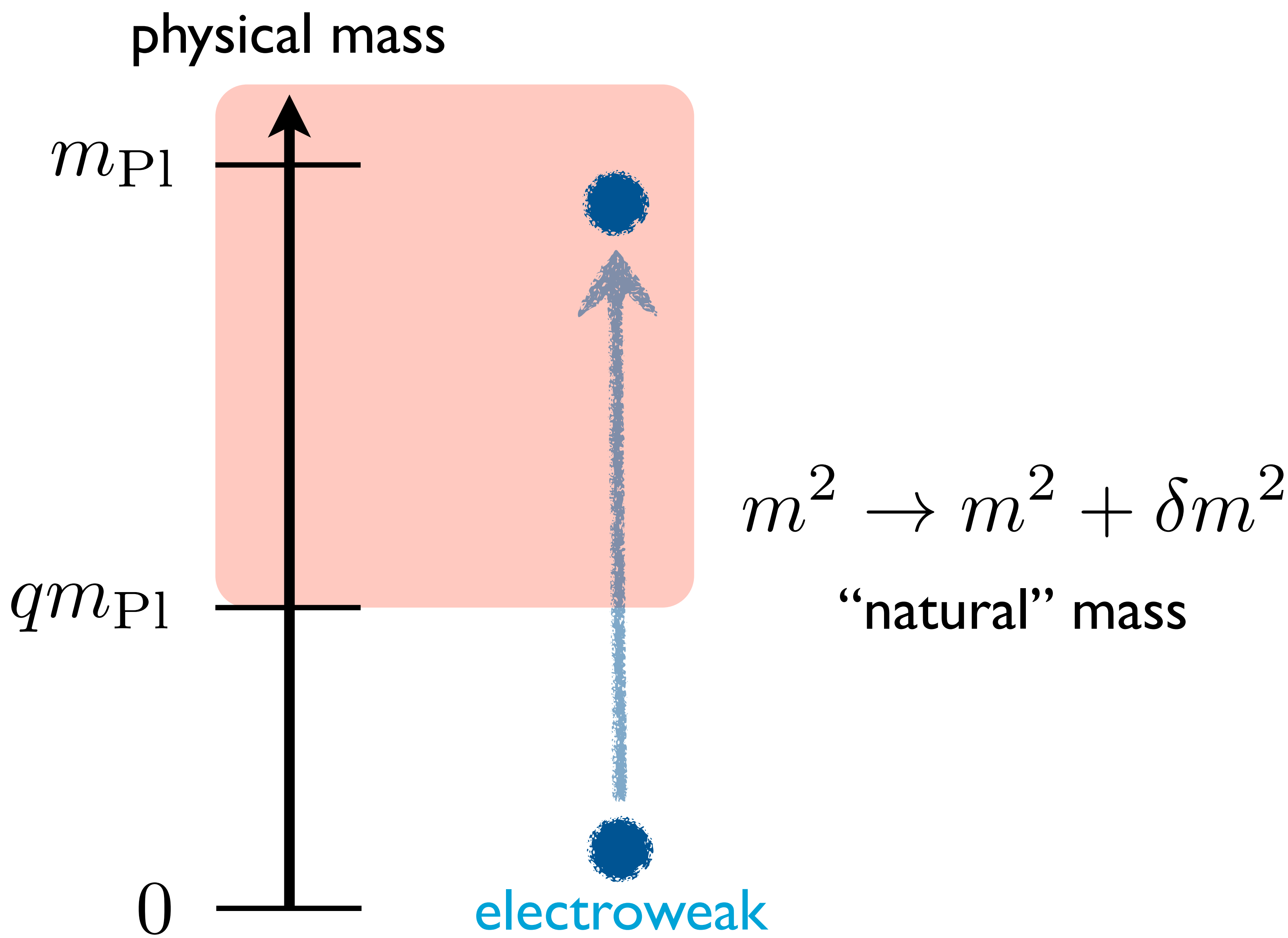

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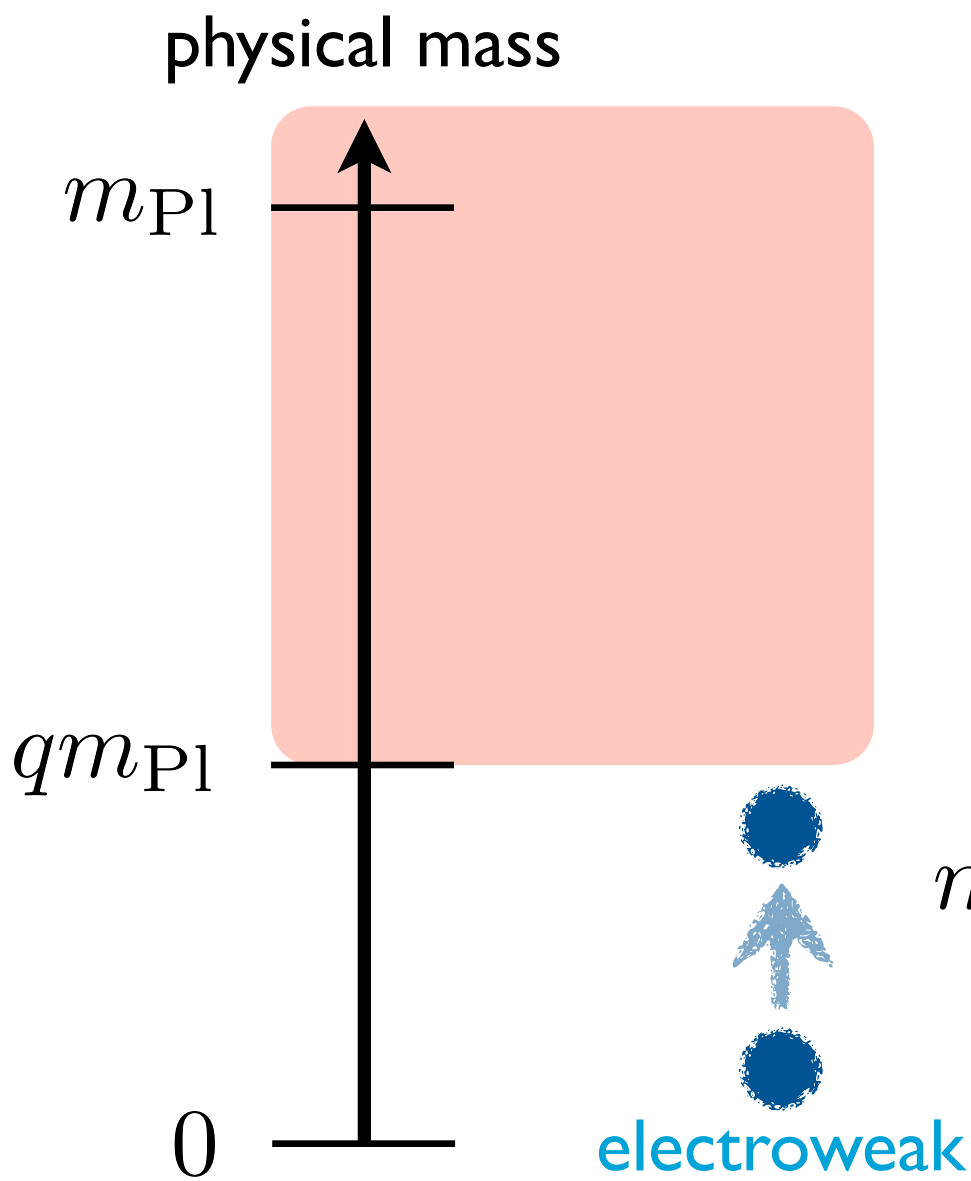

$$m^2 \to m^2 + \delta m^2$$

“**un**natural” mass

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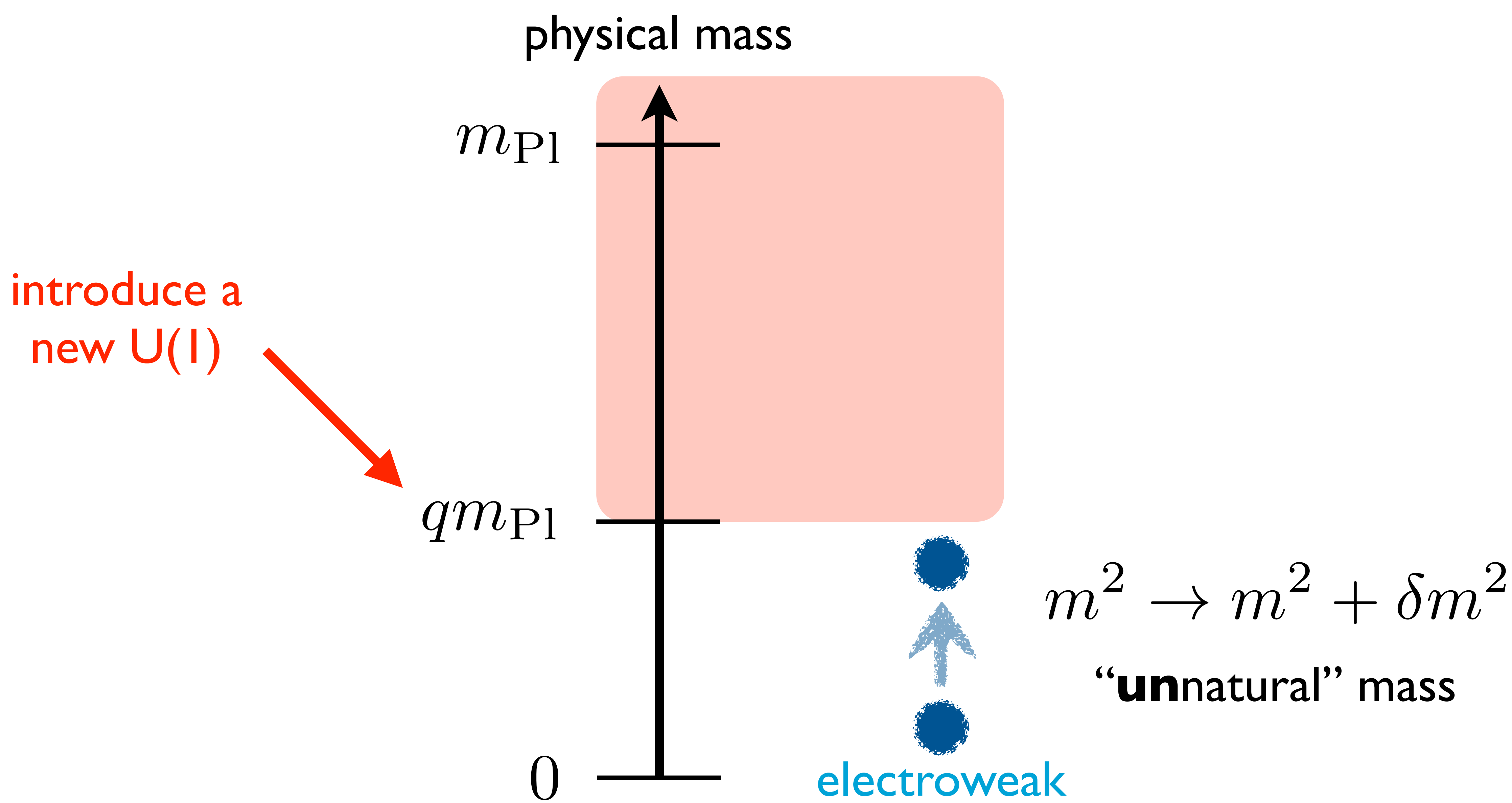

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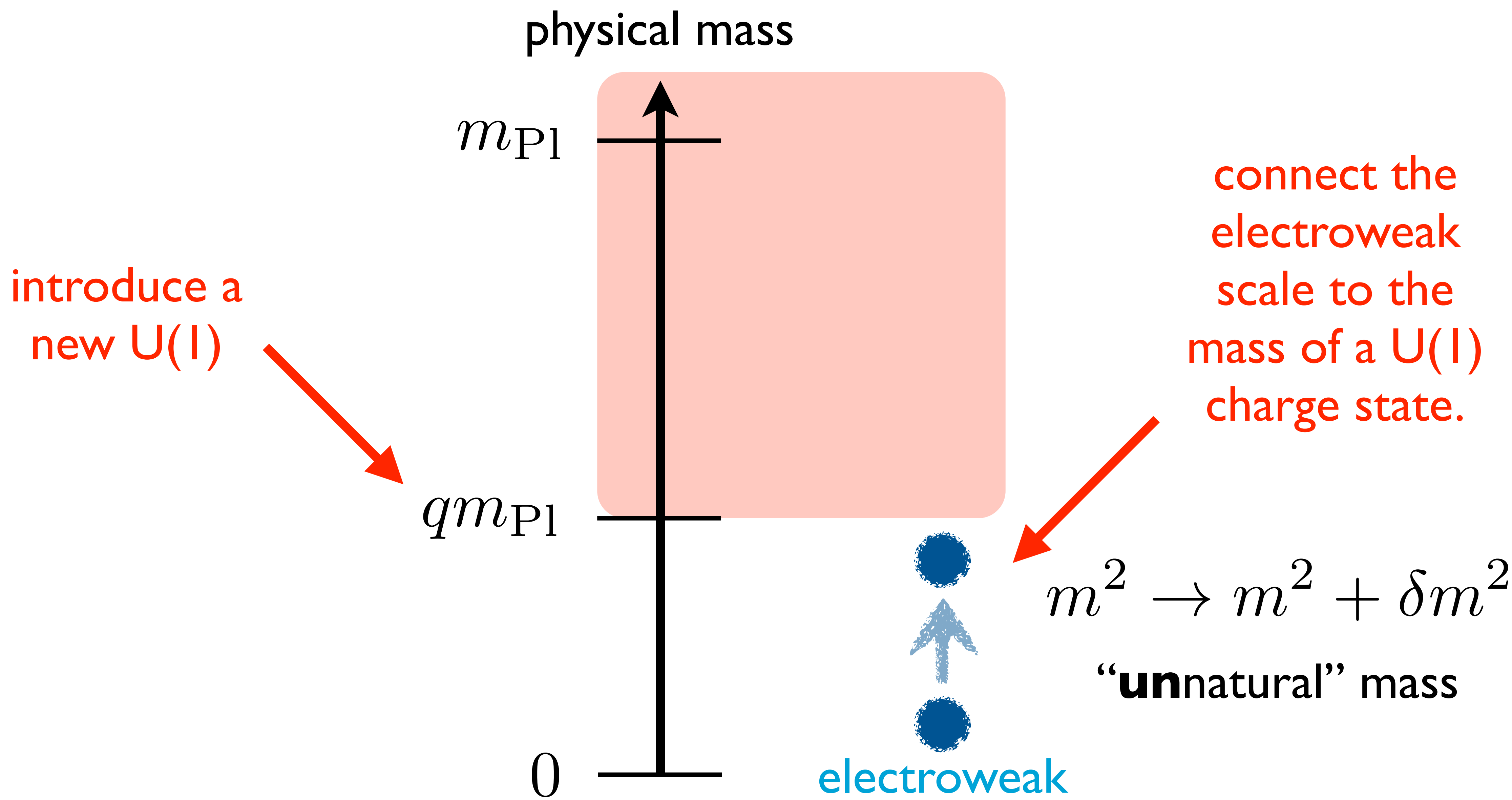
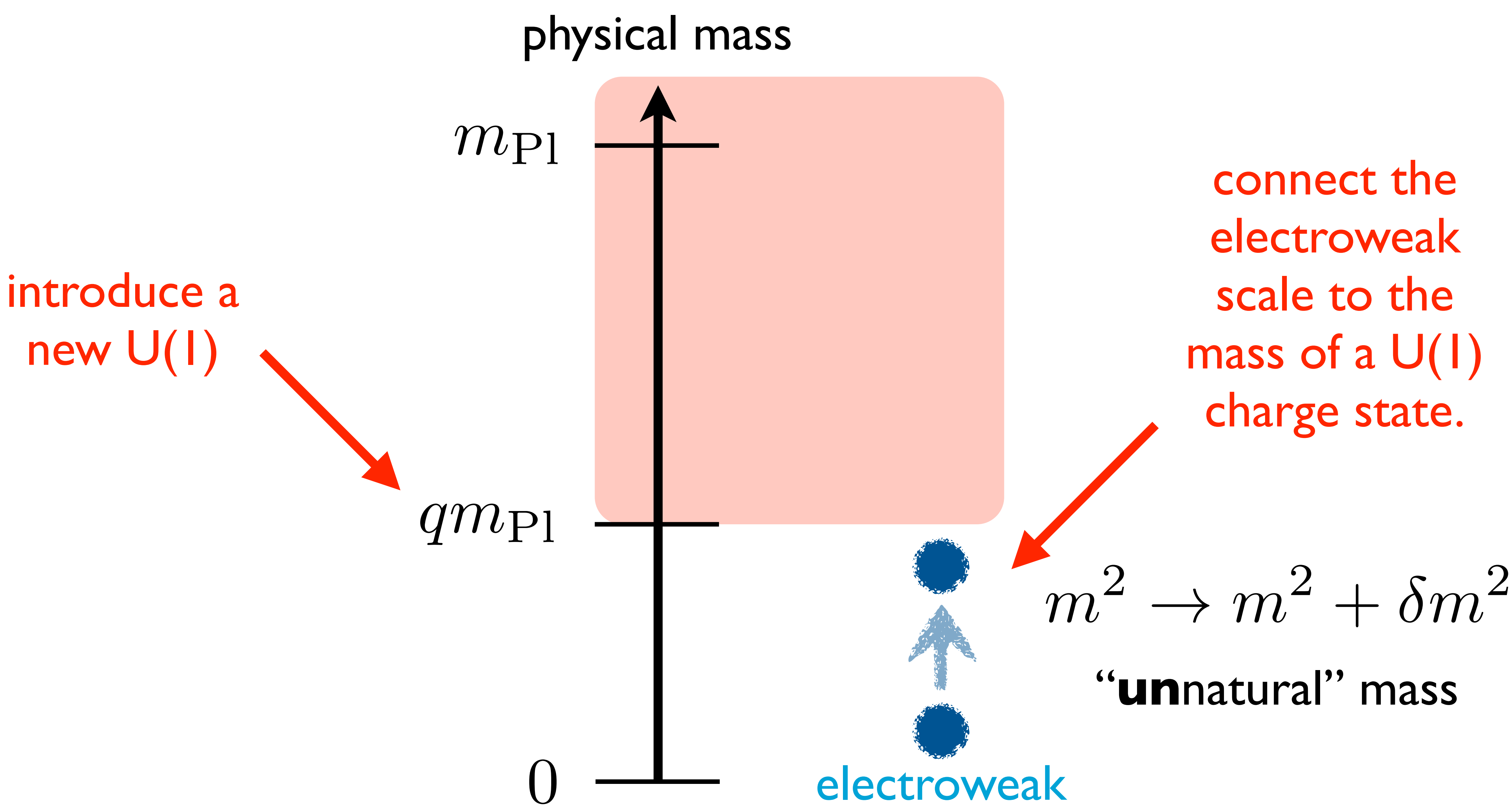


The electroweak scale is unnatural, but only because a natural value is forbidden!



model #1

Weakly gauge $U(1)_{\mathrm{B-L}}$ with Dirac neutrinos.

$$-\mathcal{L} = m_\nu \bar{\nu}_L \nu_R + \mathrm{h.c.} \qquad m_\nu \sim y_\nu v$$

Assuming that $m_\nu \sim 0.1\ \mathrm{eV}$, we fix

$$q \sim 10^{-29} \quad (\sim m_\nu / m_{\mathrm{Pl}})$$

so that the WGC is marginally satisfied.

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Weakly gauge $U(1)_{\rm B-L}$ with Dirac neutrinos.

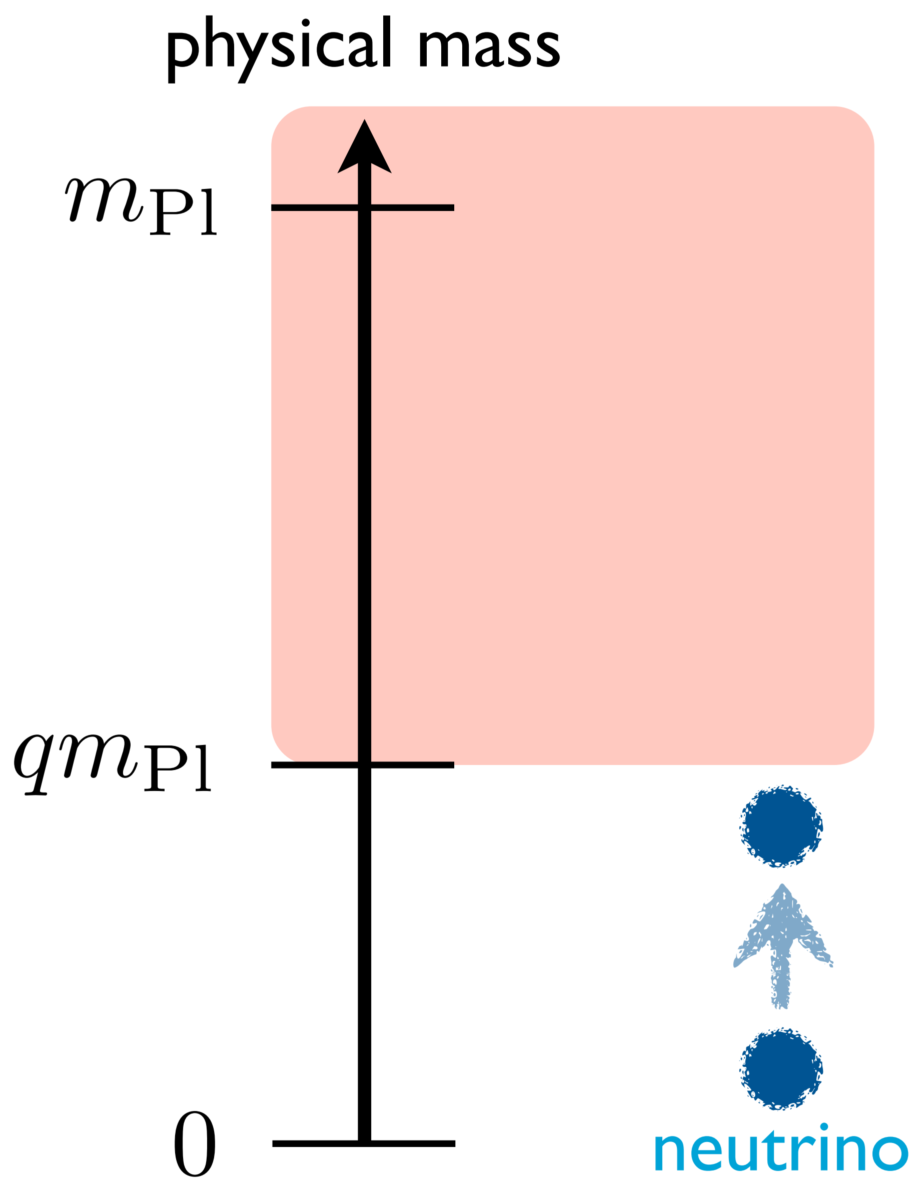
$$-\mathcal{L} = m_\nu \bar{\nu}_L \nu_R + \text{h.c.} \qquad m_\nu \sim y_\nu v$$

Assuming that $m_\nu \sim 0.1\ \text{eV}$, we fix

(technically natural) $$q \sim 10^{-29} \quad (\sim m_\nu / m_{\rm Pl})$$

so that the WGC is marginally satisfied.

Fixing couplings, were the electroweak scale larger, then the WGC condition would fail.



The model is a proof of concept but it has has a prediction: a massless gauge boson.

There are very stringent limits of fifth forces and violation of equivalence principle:

$$q \lesssim 10^{-24} \quad \text{(torsion balance)}$$

The model may yet be probed in the future.

model #2

Milli-charge dark matter under $U(1)_X$.

$$-\mathcal{L} = \frac{1}{2} m_X^2 X^2 \qquad\qquad m_X^2 \sim \lambda_X v^2 + \ldots$$

Assuming that $m_X \sim 100\ \mathrm{GeV}$, we fix

$$q \sim 10^{-16} \qquad (\sim m_X / m_{\mathrm{Pl}})$$

so that the WGC is marginally satisfied.

conclusions

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→ Beware of little (and big!) hierarchies!

thanks!